



AP[®] Calculus AB 2003 Sample Student Responses Form B

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Work for problem 2(a)

$$\int_0^{12} \left(2 + \frac{10}{1 + \ln(t+1)} \right) dt$$

$$= 70.571 \text{ gallons}$$

Work for problem 2(b)

$$H(6) = 2 + \frac{10}{1 + \ln(7)}$$

$$= 5.395 \text{ gallons coming in}$$

$$R(6) = 12 \sin\left(\frac{6^2}{47}\right)$$

$$= 8.319 \text{ gallons being removed}$$

The level of heating oil is falling at $t=6$ hours b/c more gallons are being removed than pumped into the tank. $8.319 > 5.395$ or $H(6) < R(6)$ thus lowering the level of heating oil in the tank.

Continue problem 2 on page 7.

Work for problem 2(c)

$$\int_0^{12} \left(2 + \frac{10}{1 + \ln(t+1)} \right) dt = 70.571 \text{ gallons pumped in}$$

$$\int_0^{12} \left(12 \sin\left(\frac{t^2}{47}\right) \right) dt = 73.545 \text{ gallons removed}$$

125 gallons originally

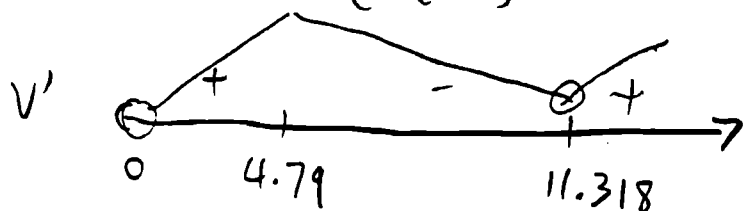
$$125 + 70.571 - 73.545 = 122.026 \text{ gallons}$$

Work for problem 2(d)

$$V = 125 + \int_0^{12} \left(2 + \frac{10}{1 + \ln(t+1)} \right) dt - \int_0^{12} \left(12 \sin\left(\frac{t^2}{47}\right) \right) dt$$

$$\frac{dV}{dt} = 2 + \frac{10}{1 + \ln(t+1)} - 12 \sin\left(\frac{t^2}{47}\right)$$

$$0 = 2 + \frac{10}{1 + \ln(t+1)} - 12 \sin\left(\frac{t^2}{47}\right)$$



$$V(11.318) = 125 + 58.118 - 58.207 = 124.917$$

$$V(0) = 125$$

Minimum volume at $t = 11.318$. Two local min at $t = 0$ and $t = 11.318$. $V(0) > V(11.318)$ so $t = 11.318$ is when volume of heating oil the least.

GO ON TO THE NEXT PAGE.

Work for problem 2(a)

$$H(t) = 2 + \frac{10}{1 + \ln(t+1)}$$

$$\int_0^{12} 2 + \frac{10}{1 + \ln(t+1)} dt$$

$$= 70.571$$

70.571 gallons of heating oil are pumped in after 12 hours.

Work for problem 2(b)

Adding oil at $2 + \frac{10}{1 + \ln(t+1)}$

extracting oil at $12 \sin\left(\frac{t^2}{47}\right)$

$$\text{Net Oil } \Delta = \left[2 + \frac{10}{1 + \ln(t+1)} \right] - \left[12 \sin\left(\frac{t^2}{47}\right) \right]$$

↑
O(x)

$$O(6) = 8.594$$

The level of the tank is rising at $t=6$.

for we have been given the rates of change for the oil in the tank. AS long as

$O(t)$ is positive oil is being added to the tank. We have been basically been given the derivative for the amount of oil in the tank.

Continue problem 2 on page 7.

Work for problem 2(c)

At $t=12$ hours there will be:

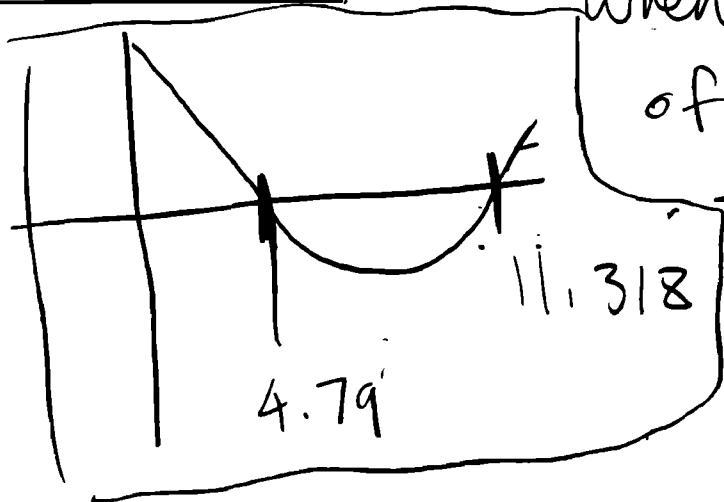
$\int_0^{12} O(t) + 125$ gallons of oil in the tank

$$Vol = \int_0^{12} \left[2 + \frac{10}{1 + \ln(t+1)} - \left[12 \sin\left(\frac{t^2}{47}\right) \right] \right] + 125$$

$$Vol = -2.974 + 125$$

Volume in tank after 12 hours = 122.026 gallons

Work for problem 2(d)



When the derivative (Rate of change of the amount of oil in the tank) is at 0, that is when there is a max and a min in the volume in the tank. If there is

a sign change from positive to negative it is a max. If it is a change from negative to positive it is a min. Therefore $O'(t) = 0$, at two different t values between 0 and 12: 4.79 and 11.318. At 11.318 there is a

sign change from negative to positive so that is the location of the minimum point. The volume of oil in the tank at $t=11.318$ hours is the least amount of oil in the tank.

GO ON TO THE NEXT PAGE.