AP® Calculus AB
2003 Sample Student Responses
Form B

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CALCULUS AB
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.

![Graph](image)

points: \( f(x) = (3, 9) \)
\( l = (3, 9) \)

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Work for problem 1(a)

Value for \( f(x) \) at \( x = 3 \): \( 4x^2 - x^3 \)

\( f(3) = 27 \)

Value for \( l \) at \( x = 3 \): \( 18 - 3x \)

\( l(3) = 9 \)

\[ \text{slope of } f'(3) \leq \text{slope of } l = -3 \]

\[ f'(3) = 8x - 3x^2 \]

\( 24 - 27 \)

\( = -3 \)

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Continue problem 1 on page 5.
Work for problem 1(b) 

\[ f(x) = 0 = 4x^2 - x^3 \]

\[ 4 - x = 0 \]

\[ x = 4 \]

\[ \left[ \int_{3}^{6} (18 - 3x) \, dx \right] - \left[ \int_{3}^{4} 4x^2 - x^3 \, dx \right] \]

\[ \left\{ \left[ 18x - \frac{3}{2} x^2 \right]_{3}^{6} \right\} - \left\{ \left[ \frac{4}{3} x^3 - \frac{1}{4} x^4 \right]_{3}^{4} \right\} \]

\[ = \left[ (54 - 40.5 \cdot 3) - (21.3 - 15.75 \cdot 3) \right] \]

\[ = \left[ (93.5 - 15.83) \right] = 79.67 \text{ units}^2 \]

Work for problem 1(c) 

\[ \pi \int_{0}^{4} (4x^2 - x^3)^2 \]

\[ = \pi \int_{0}^{4} 16x^4 - 8x^5 + x^6 \]

\[ = \pi \left[ \frac{16}{5} x^5 - \frac{8}{6} x^6 + \frac{1}{7} x^7 \right]_{0}^{4} \]

\[ = \pi \left( 156.038 \right) \]

\[ = 490.208 \text{ units}^3 \]

GO ON TO THE NEXT PAGE.
CALCULUS AB
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.

Work for problem 1(a)

Both equations have to have same value at 3.

\[ f(x) = 4x^2 - x^3 \quad y = 3 \]
\[ f(3) = 4(3)^2 - 3^3 \]
\[ = 36 - 27 \]
\[ = 9 \quad \text{when} \quad x = 3, \quad y = 9 \]

\[ y = 18 - 3x \quad x = 3 \]
\[ y = 18 - 3(3) \]
\[ y = 18 - 9 \]
\[ = 9 \quad \text{when} \quad x = 3, \quad y = 9 \]

So \( d \) is tangent to the graph of \( y = f(x) \) at the point \( x = 3 \).

Continue problem 1 on page 5.
Work for problem 1(b)

\[ f(x) = 4x^2 - x^3 \]
\[ 0 = 4x^2 - x^3 \]
\[ = x^2(4 - x) \]
\[ x = 0, 4 \]

\[ y = 18 - 3x \]
\[ 0 = 18 - 3x \]
\[ -18 = -3x \]
\[ x = 6 \]

\[ S = \int_3^4 (18 - 3x) - (4x^2 - x^3) \, dx + \int_4^6 (18 - 3x) \, dx \]
\[ = 1.917 + 6 \]
\[ = 7.917 \]

Work for problem 1(c)

\[ V = \pi \int_0^4 (4x^2 - x^3)^2 \, dx \]
\[ = \pi \cdot 156.04 \]
\[ = 490.21 \]