



AP[®] Calculus AB 2007 Scoring Guidelines

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Question 1

Let R be the region in the first and second quadrants bounded above by the graph of $y = \frac{20}{1+x^2}$ and below by the horizontal line $y = 2$.

- (a) Find the area of R .
- (b) Find the volume of the solid generated when R is rotated about the x -axis.
- (c) The region R is the base of a solid. For this solid, the cross sections perpendicular to the x -axis are semicircles. Find the volume of this solid.

$$\frac{20}{1+x^2} = 2 \text{ when } x = \pm 3$$

(a) $\text{Area} = \int_{-3}^3 \left(\frac{20}{1+x^2} - 2 \right) dx = 37.961 \text{ or } 37.962$

(b) $\text{Volume} = \pi \int_{-3}^3 \left(\left(\frac{20}{1+x^2} \right)^2 - 2^2 \right) dx = 1871.190$

(c) $\text{Volume} = \frac{\pi}{2} \int_{-3}^3 \left(\frac{1}{2} \left(\frac{20}{1+x^2} - 2 \right) \right)^2 dx$
 $= \frac{\pi}{8} \int_{-3}^3 \left(\frac{20}{1+x^2} - 2 \right)^2 dx = 174.268$

1 : correct limits in an integral in
(a), (b), or (c)

2 : $\begin{cases} 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

3 : $\begin{cases} 2 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

3 : $\begin{cases} 2 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

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Question 2

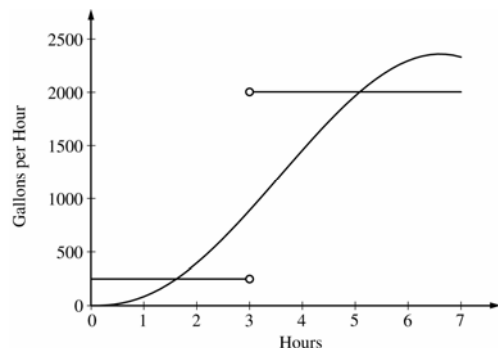
The amount of water in a storage tank, in gallons, is modeled by a continuous function on the time interval $0 \leq t \leq 7$, where t is measured in hours. In this model, rates are given as follows:

(i) The rate at which water enters the tank is

$$f(t) = 100t^2 \sin(\sqrt{t}) \text{ gallons per hour for } 0 \leq t \leq 7.$$

(ii) The rate at which water leaves the tank is

$$g(t) = \begin{cases} 250 & \text{for } 0 \leq t < 3 \\ 2000 & \text{for } 3 < t \leq 7 \end{cases} \text{ gallons per hour.}$$



The graphs of f and g , which intersect at $t = 1.617$ and $t = 5.076$, are shown in the figure above. At time $t = 0$, the amount of water in the tank is 5000 gallons.

- (a) How many gallons of water enter the tank during the time interval $0 \leq t \leq 7$? Round your answer to the nearest gallon.
- (b) For $0 \leq t \leq 7$, find the time intervals during which the amount of water in the tank is decreasing. Give a reason for each answer.
- (c) For $0 \leq t \leq 7$, at what time t is the amount of water in the tank greatest? To the nearest gallon, compute the amount of water at this time. Justify your answer.

(a) $\int_0^7 f(t) dt \approx 8264$ gallons

2 : $\begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(b) The amount of water in the tank is decreasing on the intervals $0 \leq t \leq 1.617$ and $3 \leq t \leq 5.076$ because $f(t) < g(t)$ for $0 \leq t < 1.617$ and $3 < t < 5.076$.

2 : $\begin{cases} 1 : \text{intervals} \\ 1 : \text{reason} \end{cases}$

(c) Since $f(t) - g(t)$ changes sign from positive to negative only at $t = 3$, the candidates for the absolute maximum are at $t = 0, 3$, and 7 .

5 : $\begin{cases} 1 : \text{identifies } t = 3 \text{ as a candidate} \\ 1 : \text{integrand} \\ 1 : \text{amount of water at } t = 3 \\ 1 : \text{amount of water at } t = 7 \\ 1 : \text{conclusion} \end{cases}$

t (hours)	gallons of water
0	5000
3	$5000 + \int_0^3 f(t) dt - 250(3) = 5126.591$
7	$5126.591 + \int_3^7 f(t) dt - 2000(4) = 4513.807$

The amount of water in the tank is greatest at 3 hours. At that time, the amount of water in the tank, rounded to the nearest gallon, is 5127 gallons.

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Question 3

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	6	4	2	5
2	9	2	3	1
3	10	-4	4	2
4	-1	3	6	7

The functions f and g are differentiable for all real numbers, and g is strictly increasing. The table above gives values of the functions and their first derivatives at selected values of x . The function h is given by $h(x) = f(g(x)) - 6$.

- (a) Explain why there must be a value r for $1 < r < 3$ such that $h(r) = -5$.
- (b) Explain why there must be a value c for $1 < c < 3$ such that $h'(c) = -5$.
- (c) Let w be the function given by $w(x) = \int_1^{g(x)} f(t) dt$. Find the value of $w'(3)$.
- (d) If g^{-1} is the inverse function of g , write an equation for the line tangent to the graph of $y = g^{-1}(x)$ at $x = 2$.

(a) $h(1) = f(g(1)) - 6 = f(2) - 6 = 9 - 6 = 3$
 $h(3) = f(g(3)) - 6 = f(4) - 6 = -1 - 6 = -7$
 Since $h(3) < -5 < h(1)$ and h is continuous, by the Intermediate Value Theorem, there exists a value r , $1 < r < 3$, such that $h(r) = -5$.

2 : $\left\{ \begin{array}{l} 1 : h(1) \text{ and } h(3) \\ 1 : \text{conclusion, using IVT} \end{array} \right.$

(b) $\frac{h(3) - h(1)}{3 - 1} = \frac{-7 - 3}{3 - 1} = -5$
 Since h is continuous and differentiable, by the Mean Value Theorem, there exists a value c , $1 < c < 3$, such that $h'(c) = -5$.

2 : $\left\{ \begin{array}{l} 1 : \frac{h(3) - h(1)}{3 - 1} \\ 1 : \text{conclusion, using MVT} \end{array} \right.$

(c) $w'(3) = f(g(3)) \cdot g'(3) = f(4) \cdot 2 = -2$

2 : $\left\{ \begin{array}{l} 1 : \text{apply chain rule} \\ 1 : \text{answer} \end{array} \right.$

(d) $g(1) = 2$, so $g^{-1}(2) = 1$.
 $(g^{-1})'(2) = \frac{1}{g'(g^{-1}(2))} = \frac{1}{g'(1)} = \frac{1}{5}$

3 : $\left\{ \begin{array}{l} 1 : g^{-1}(2) \\ 1 : (g^{-1})'(2) \\ 1 : \text{tangent line equation} \end{array} \right.$

An equation of the tangent line is $y - 1 = \frac{1}{5}(x - 2)$.

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Question 4

A particle moves along the x -axis with position at time t given by $x(t) = e^{-t} \sin t$ for $0 \leq t \leq 2\pi$.

- (a) Find the time t at which the particle is farthest to the left. Justify your answer.
 (b) Find the value of the constant A for which $x(t)$ satisfies the equation $Ax''(t) + x'(t) + x(t) = 0$ for $0 < t < 2\pi$.

- (a) $x'(t) = -e^{-t} \sin t + e^{-t} \cos t = e^{-t} (\cos t - \sin t)$
 $x'(t) = 0$ when $\cos t = \sin t$. Therefore, $x'(t) = 0$ on
 $0 \leq t \leq 2\pi$ for $t = \frac{\pi}{4}$ and $t = \frac{5\pi}{4}$.
 The candidates for the absolute minimum are at
 $t = 0, \frac{\pi}{4}, \frac{5\pi}{4}$, and 2π .

t	$x(t)$
0	$e^0 \sin(0) = 0$
$\frac{\pi}{4}$	$e^{-\frac{\pi}{4}} \sin\left(\frac{\pi}{4}\right) > 0$
$\frac{5\pi}{4}$	$e^{-\frac{5\pi}{4}} \sin\left(\frac{5\pi}{4}\right) < 0$
2π	$e^{-2\pi} \sin(2\pi) = 0$

The particle is farthest to the left when $t = \frac{5\pi}{4}$.

- (b) $x''(t) = -e^{-t} (\cos t - \sin t) + e^{-t} (-\sin t - \cos t)$
 $= -2e^{-t} \cos t$
 $Ax''(t) + x'(t) + x(t)$
 $= A(-2e^{-t} \cos t) + e^{-t} (\cos t - \sin t) + e^{-t} \sin t$
 $= (-2A + 1)e^{-t} \cos t$
 $= 0$
 Therefore, $A = \frac{1}{2}$.

5 : $\left\{ \begin{array}{l} 2 : x'(t) \\ 1 : \text{sets } x'(t) = 0 \\ 1 : \text{answer} \\ 1 : \text{justification} \end{array} \right.$

4 : $\left\{ \begin{array}{l} 2 : x''(t) \\ 1 : \text{substitutes } x''(t), x'(t), \text{ and } x(t) \\ \quad \text{into } Ax''(t) + x'(t) + x(t) \\ 1 : \text{answer} \end{array} \right.$

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Question 5

t (minutes)	0	2	5	7	11	12
$r'(t)$ (feet per minute)	5.7	4.0	2.0	1.2	0.6	0.5

The volume of a spherical hot air balloon expands as the air inside the balloon is heated. The radius of the balloon, in feet, is modeled by a twice-differentiable function r of time t , where t is measured in minutes. For $0 < t < 12$, the graph of r is concave down. The table above gives selected values of the rate of change, $r'(t)$, of the radius of the balloon over the time interval $0 \leq t \leq 12$. The radius of the balloon is 30 feet when $t = 5$. (Note: The volume of a sphere of radius r is given by $V = \frac{4}{3}\pi r^3$.)

- (a) Estimate the radius of the balloon when $t = 5.4$ using the tangent line approximation at $t = 5$. Is your estimate greater than or less than the true value? Give a reason for your answer.
- (b) Find the rate of change of the volume of the balloon with respect to time when $t = 5$. Indicate units of measure.
- (c) Use a right Riemann sum with the five subintervals indicated by the data in the table to approximate $\int_0^{12} r'(t) dt$. Using correct units, explain the meaning of $\int_0^{12} r'(t) dt$ in terms of the radius of the balloon.
- (d) Is your approximation in part (c) greater than or less than $\int_0^{12} r'(t) dt$? Give a reason for your answer.

(a) $r(5.4) \approx r(5) + r'(5)\Delta t = 30 + 2(0.4) = 30.8$ ft
Since the graph of r is concave down on the interval $5 < t < 5.4$, this estimate is greater than $r(5.4)$.

(b) $\frac{dV}{dt} = 3\left(\frac{4}{3}\right)\pi r^2 \frac{dr}{dt}$
 $\left.\frac{dV}{dt}\right|_{t=5} = 4\pi(30)^2 2 = 7200\pi$ ft³/min

(c) $\int_0^{12} r'(t) dt \approx 2(4.0) + 3(2.0) + 2(1.2) + 4(0.6) + 1(0.5)$
 $= 19.3$ ft
 $\int_0^{12} r'(t) dt$ is the change in the radius, in feet, from $t = 0$ to $t = 12$ minutes.

(d) Since r is concave down, r' is decreasing on $0 < t < 12$. Therefore, this approximation, 19.3 ft, is less than $\int_0^{12} r'(t) dt$.

Units of ft³/min in part (b) and ft in part (c)

2 : $\begin{cases} 1 : \text{estimate} \\ 1 : \text{conclusion with reason} \end{cases}$

3 : $\begin{cases} 2 : \frac{dV}{dt} \\ 1 : \text{answer} \end{cases}$

2 : $\begin{cases} 1 : \text{approximation} \\ 1 : \text{explanation} \end{cases}$

1 : conclusion with reason

1 : units in (b) and (c)

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Question 6

Let f be the function defined by $f(x) = k\sqrt{x} - \ln x$ for $x > 0$, where k is a positive constant.

- (a) Find $f'(x)$ and $f''(x)$.
- (b) For what value of the constant k does f have a critical point at $x = 1$? For this value of k , determine whether f has a relative minimum, relative maximum, or neither at $x = 1$. Justify your answer.
- (c) For a certain value of the constant k , the graph of f has a point of inflection on the x -axis. Find this value of k .

(a) $f'(x) = \frac{k}{2\sqrt{x}} - \frac{1}{x}$

$$f''(x) = -\frac{1}{4}kx^{-3/2} + x^{-2}$$

$$2 : \begin{cases} 1 : f'(x) \\ 1 : f''(x) \end{cases}$$

(b) $f'(1) = \frac{1}{2}k - 1 = 0 \Rightarrow k = 2$

When $k = 2$, $f'(1) = 0$ and $f''(1) = -\frac{1}{2} + 1 > 0$.

f has a relative minimum value at $x = 1$ by the Second Derivative Test.

$$4 : \begin{cases} 1 : \text{sets } f'(1) = 0 \text{ or } f'(x) = 0 \\ 1 : \text{solves for } k \\ 1 : \text{answer} \\ 1 : \text{justification} \end{cases}$$

(c) At this inflection point, $f''(x) = 0$ and $f(x) = 0$.

$$f''(x) = 0 \Rightarrow \frac{-k}{4x^{3/2}} + \frac{1}{x^2} = 0 \Rightarrow k = \frac{4}{\sqrt{x}}$$

$$f(x) = 0 \Rightarrow k\sqrt{x} - \ln x = 0 \Rightarrow k = \frac{\ln x}{\sqrt{x}}$$

Therefore, $\frac{4}{\sqrt{x}} = \frac{\ln x}{\sqrt{x}}$
 $\Rightarrow 4 = \ln x$
 $\Rightarrow x = e^4$
 $\Rightarrow k = \frac{4}{e^2}$

$$3 : \begin{cases} 1 : f''(x) = 0 \text{ or } f(x) = 0 \\ 1 : \text{equation in one variable} \\ 1 : \text{answer} \end{cases}$$