AP® Physics C: Mechanics
2005 Free-Response Questions

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TABLE OF INFORMATION FOR 2005

CONSTANTS AND CONVERSION FACTORS

1 unified atomic mass unit,	$1\ \text{u} = 1.66 \times 10^{-27}\ \text{kg}$
	$= 931\ \text{MeV}/c^2$
Proton mass,	$m_p = 1.67 \times 10^{-27}\ \text{kg}$
Neutron mass,	$m_n = 1.67 \times 10^{-27}\ \text{kg}$
Electron mass,	$m_e = 9.11 \times 10^{-31}\ \text{kg}$
Magnitude of the electron charge,	$e = 1.60 \times 10^{-19}\ \text{C}$
Avogadro's number,	$N_0 = 6.02 \times 10^{23}\ \text{mol}^{-1}$
Universal gas constant,	$R = 8.31\ \text{J}/(\text{mol} \cdot \text{K})$
Boltzmann's constant,	$k_B = 1.38 \times 10^{-23}\ \text{J}/\text{K}$
Speed of light,	$c = 3.00 \times 10^{8}\ \text{m}/\text{s}$
Planck's constant,	$h = 6.63 \times 10^{-34}\ \text{J} \cdot \text{s}$
	$= 4.14 \times 10^{-15}\ \text{eV} \cdot \text{s}$
	$hc = 1.99 \times 10^{-25}\ \text{J} \cdot \text{m}$
	$= 1.24 \times 10^{3}\ \text{eV} \cdot \text{nm}$
Vacuum permittivity,	$\epsilon_0 = 8.85 \times 10^{-12}\ \text{C}^2/\text{N} \cdot \text{m}^2$
Coulomb's law constant,	$k = 1/4\pi\epsilon_0 = 9.0 \times 10^{9}\ \text{N} \cdot \text{m}^2/\text{C}^2$
Vacuum permeability,	$\mu_0 = 4\pi \times 10^{-7}(\text{T} \cdot \text{m})/\text{A}$
Magnetic constant,	$k' = \mu_0/4\pi = 10^{-7}(\text{T} \cdot \text{m})/\text{A}$
Universal gravitational constant,	$G = 6.67 \times 10^{-11}\ \text{m}^3/\text{kg} \cdot \text{s}^2$
Acceleration due to gravity at the Earth's surface,	$g = 9.8\ \text{m}/\text{s}^2$
1 atmosphere pressure,	$1\ \text{atm} = 1.0 \times 10^{5}\ \text{N}/\text{m}^2$
	$= 1.0 \times 10^{5}\ \text{Pa}$
1 electron volt,	$1\ \text{eV} = 1.60 \times 10^{-19}\ \text{J}$

UNITS

Name	Symbol
meter	m
kilogram	kg
second	s
ampere	A
kelvin	K
mole	mol
hertz	Hz
newton	N
pascal	Pa
joule	J
watt	W
coulomb	C
volt	V
ohm	Ω
henry	H
farad	F
tesla	T
degree Celsius	°C
electron-volt	eV

PREFIXES

Factor	Prefix	Symbol
10^{9}	giga	G
10^{6}	mega	M
10^{3}	kilo	k
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p

VALUES OF TRIGONOMETRIC FUNCTIONS FOR COMMON ANGLES

θ	sin θ	cos θ	tan θ
0°	0	1	0
30°	1/2	$\sqrt{3}/2$	$\sqrt{3}/3$
37°	3/5	4/5	3/4
45°	$\sqrt{2}/2$	$\sqrt{2}/2$	1
53°	4/5	3/5	4/3
60°	$\sqrt{3}/2$	1/2	$\sqrt{3}$
90°	1	0	∞

The following conventions are used in this examination.

I. Unless otherwise stated, the frame of reference of any problem is assumed to be inertial.

II. The direction of any electric current is the direction of flow of positive charge (conventional current).

III. For any isolated electric charge, the electric potential is defined as zero at an infinite distance from the charge.

ADVANCED PLACEMENT PHYSICS C EQUATIONS FOR 2004 and 2005

MECHANICS

$v = v_0 + at$

$x = x_0 + v_0 t + \frac{1}{2}at^2$

$v^2 = v_0^2 + 2a(x - x_0)$

$\sum \mathbf{F} = \mathbf{F}_{net} = m\mathbf{a}$

$\mathbf{F} = \frac{d\mathbf{p}}{dt}$

$\mathbf{J} = \int \mathbf{F}\,dt = \Delta\mathbf{p}$

$\mathbf{p} = m\mathbf{v}$

$F_{fric} \le \mu N$

$W = \int \mathbf{F} \cdot d\mathbf{r}$

$K = \frac{1}{2}mv^2$

$P = \frac{dW}{dt}$

$P = \mathbf{F} \cdot \mathbf{v}$

$\Delta U_g = mgh$

$a_c = \frac{v^2}{r} = \omega^2 r$

$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$

$\sum \boldsymbol{\tau} = \boldsymbol{\tau}_{net} = I\boldsymbol{\alpha}$

$I = \int r^2 dm = \sum mr^2$

$\mathbf{r}_{cm} = \sum m\mathbf{r} / \sum m$

$v = r\omega$

$\mathbf{L} = \mathbf{r} \times \mathbf{p} = I\boldsymbol{\omega}$

$K = \frac{1}{2}I\omega^2$

$\omega = \omega_0 + \alpha t$

$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$

$\mathbf{F}_s = -k\mathbf{x}$

$U_s = \frac{1}{2}kx^2$

$T = \frac{2\pi}{\omega} = \frac{1}{f}$

$T_s = 2\pi\sqrt{\frac{m}{k}}$

$T_p = 2\pi\sqrt{\frac{\ell}{g}}$

$\mathbf{F}_G = -\frac{Gm_1m_2}{r^2}\hat{\mathbf{r}}$

$U_G = -\frac{Gm_1m_2}{r}$

a = acceleration
F = force
f = frequency
h = height
I = rotational inertia
J = impulse
K = kinetic energy
k = spring constant
ℓ = length
L = angular momentum
m = mass
N = normal force
P = power
p = momentum
r = radius or distance
$\mathbf{r}$ = position vector
T = period
t = time
U = potential energy
v = velocity or speed
W = work done on a system
x = position
μ = coefficient of friction
θ = angle
τ = torque
ω = angular speed
α = angular acceleration

ELECTRICITY AND MAGNETISM

$F = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r^2}$

$\mathbf{E} = \frac{\mathbf{F}}{q}$

$\oint \mathbf{E} \cdot d\mathbf{A} = \frac{Q}{\epsilon_0}$

$E = -\frac{dV}{dr}$

$V = \frac{1}{4\pi\epsilon_0}\sum_i \frac{q_i}{r_i}$

$U_E = qV = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r}$

$C = \frac{Q}{V}$

$C = \frac{\kappa\epsilon_0 A}{d}$

$C_p = \sum_i C_i$

$\frac{1}{C_s} = \sum_i \frac{1}{C_i}$

$I = \frac{dQ}{dt}$

$U_c = \frac{1}{2}QV = \frac{1}{2}CV^2$

$R = \frac{\rho\ell}{A}$

$V = IR$

$R_s = \sum_i R_i$

$\frac{1}{R_p} = \sum_i \frac{1}{R_i}$

$P = IV$

$\mathbf{F}_M = q\mathbf{v} \times \mathbf{B}$

$\oint \mathbf{B} \cdot d\boldsymbol{\ell} = \mu_0 I$

$\mathbf{F} = \int I\,d\boldsymbol{\ell} \times \mathbf{B}$

$B_s = \mu_0 nI$

$\phi_m = \int \mathbf{B} \cdot d\mathbf{A}$

$\mathcal{E} = -\frac{d\phi_m}{dt}$

$\mathcal{E} = -L\frac{dI}{dt}$

$U_L = \frac{1}{2}LI^2$

A = area
B = magnetic field
C = capacitance
d = distance
E = electric field
$\mathcal{E}$ = emf
F = force
I = current
L = inductance
ℓ = length
n = number of loops of wire per unit length
P = power
Q = charge
q = point charge
R = resistance
r = distance
t = time
U = potential or stored energy
V = electric potential
v = velocity or speed
ρ = resistivity
ϕ_m = magnetic flux
κ = dielectric constant

GEOMETRY AND TRIGONOMETRY

Rectangle
$A = bh$

Triangle
$A = \frac{1}{2} bh$

Circle
$A = \pi r^2$
$C = 2\pi r$

Parallelepiped
$V = \ell wh$

Cylinder
$V = \pi r^2 \ell$
$S = 2\pi r\ell + 2\pi r^2$

Sphere
$V = \frac{4}{3}\pi r^3$
$S = 4\pi r^2$

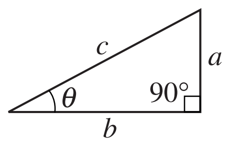
Right Triangle
$a^2 + b^2 = c^2$

$\sin\theta = \frac{a}{c}$

$\cos\theta = \frac{b}{c}$

$\tan\theta = \frac{a}{b}$

A = area
C = circumference
V = volume
S = surface area
b = base
h = height
ℓ = length
w = width
r = radius



CALCULUS

$$\frac{df}{dx} = \frac{df}{du}\frac{du}{dx}$$

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$$

$$\frac{d}{dx}\left(e^x\right) = e^x$$

$$\frac{d}{dx}(\ln x) = \frac{1}{x}$$

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

$$\int x^n dx = \frac{1}{n+1}x^{n+1},\ n \neq -1$$

$$\int e^x dx = e^x$$

$$\int \frac{dx}{x} = \ln|x|$$

$$\int \cos x\, dx = \sin x$$

$$\int \sin x\, dx = -\cos x$$

PHYSICS C
Section II, MECHANICS
Time—45 minutes
3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in the pink booklet in the spaces provided after each part, NOT in this green insert.

Mech. 1.

A ball of mass M is thrown vertically upward with an initial speed of v_0. It experiences a force of air resistance given by $\mathbf{F} = -k\mathbf{v}$, where k is a positive constant. The positive direction for all vector quantities is upward. Express all algebraic answers in terms of M, k, v_0, and fundamental constants.

(a) Does the magnitude of the acceleration of the ball increase, decrease, or remain the same as the ball moves upward?

____ increases ____ decreases ____ remains the same

Justify your answer.

(b) Write, but do NOT solve, a differential equation for the instantaneous speed v of the ball in terms of time t as the ball moves upward.

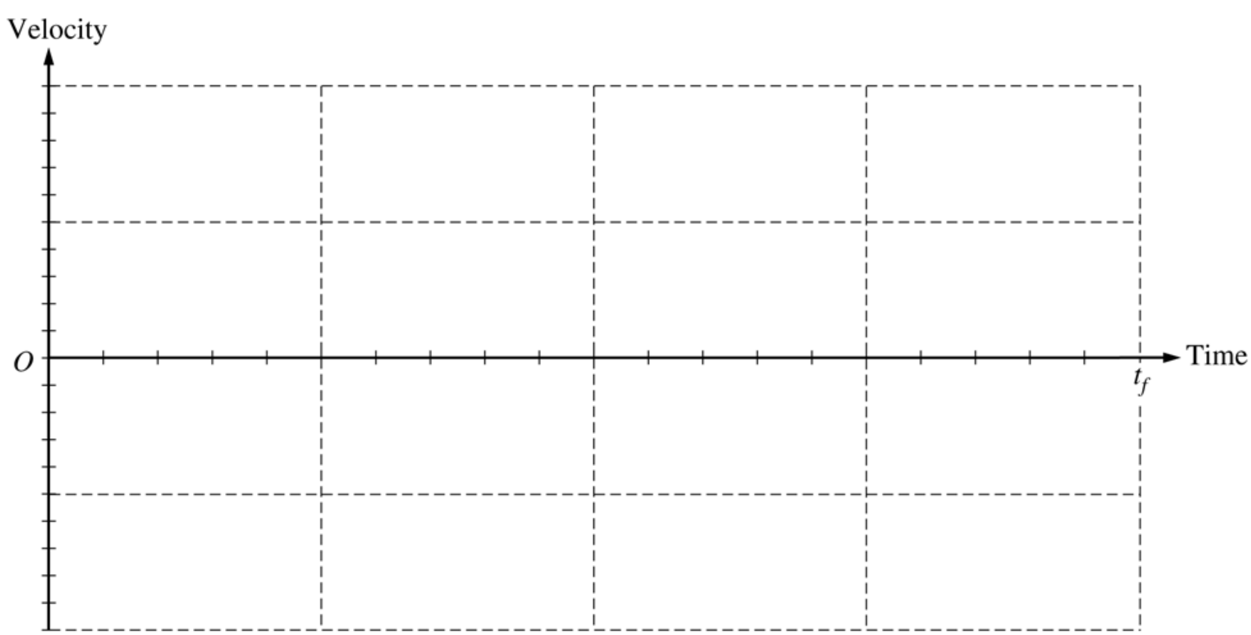
(c) Determine the terminal speed of the ball as it moves downward.

(d) Does it take longer for the ball to rise to its maximum height or to fall from its maximum height back to the height from which it was thrown?

____longer to rise ____longer to fall

Justify your answer.

(e) On the axes below, sketch a graph of velocity versus time for the upward and downward parts of the ball's flight, where t_f is the time at which the ball returns to the height from which it was thrown.



Mech. 2.

A student is given the set of orbital data for some of the moons of Saturn shown below and is asked to use the data to determine the mass M_S of Saturn. Assume the orbits of these moons are circular.

Orbital Period, T (seconds)	Orbital Radius, R (meters)		
8.14×10^4	1.85×10^8		
1.18×10^5	2.38×10^8		
1.63×10^5	2.95×10^8		
2.37×10^5	3.77×10^8		

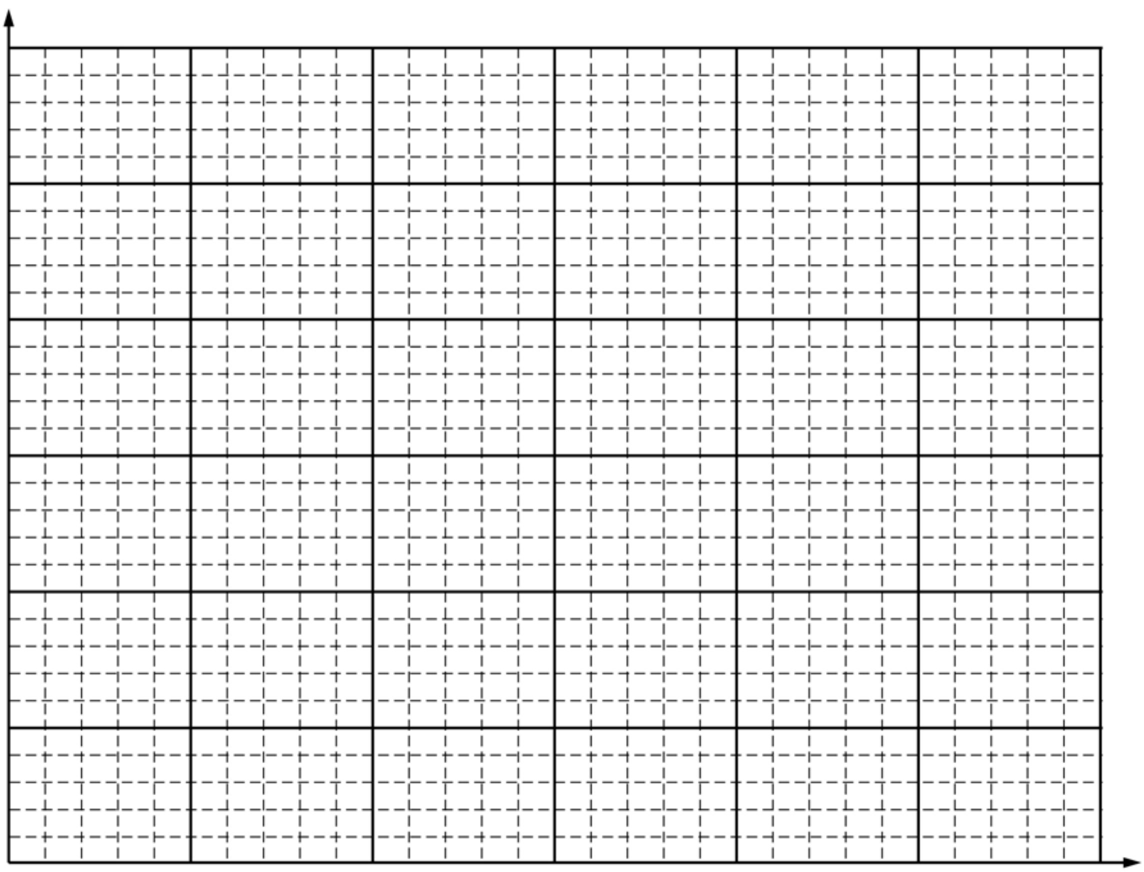
(a) Write an algebraic expression for the gravitational force between Saturn and one of its moons.

(b) Use your expression from part (a) and the assumption of circular orbits to derive an equation for the orbital period T of a moon as a function of its orbital radius R.

(c) Which quantities should be graphed to yield a straight line whose slope could be used to determine Saturn's mass?

(d) Complete the data table by calculating the two quantities to be graphed. Label the top of each column, including units.

(e) Plot the graph on the axes below. Label the axes with the variables used and appropriate numbers to indicate the scale.



(f) Using the graph, calculate a value for the mass of Saturn.

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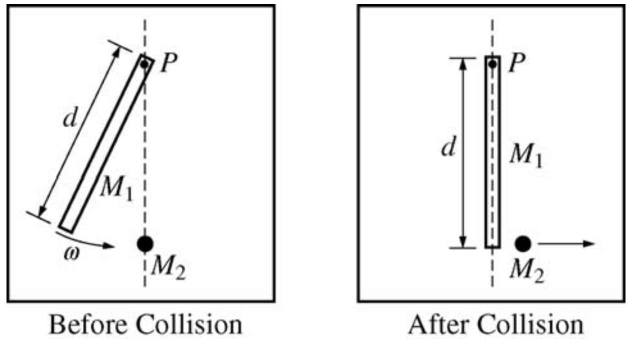


TOP VIEWS

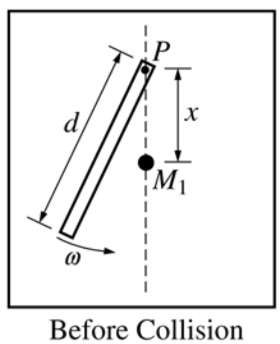
Mech. 3.

A system consists of a ball of mass M_2 and a uniform rod of mass M_1 and length d. The rod is attached to a horizontal frictionless table by a pivot at point P and initially rotates at an angular speed ω, as shown above left. The rotational inertia of the rod about point P is $\frac{1}{3}M_1d^2$. The rod strikes the ball, which is initially at rest. As a result of this collision, the rod is stopped and the ball moves in the direction shown above right. Express all answers in terms of M_1, M_2, ω, d, and fundamental constants.

(a) Derive an expression for the angular momentum of the rod about point P before the collision.

(b) Derive an expression for the speed v of the ball after the collision.

(c) Assuming that this collision is elastic, calculate the numerical value of the ratio M_1/M_2.



(d) A new ball with the same mass M_1 as the rod is now placed a distance x from the pivot, as shown above. Again assuming the collision is elastic, for what value of x will the rod stop moving after hitting the ball?

END OF SECTION II, MECHANICS