



**AP<sup>®</sup> Calculus AB**  
**2005 Free-Response Questions**  
**Form B**

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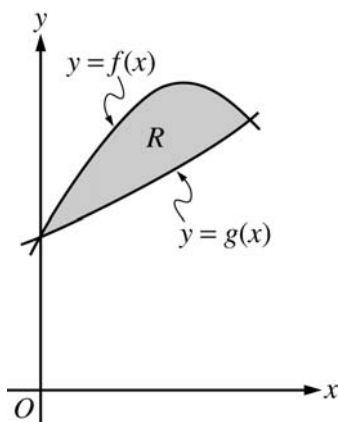
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**2005 AP<sup>®</sup> CALCULUS AB FREE-RESPONSE QUESTIONS (Form B)**

**CALCULUS AB**  
**SECTION II, Part A**  
**Time—45 minutes**  
**Number of problems—3**

A graphing calculator is required for some problems or parts of problems.

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1. Let  $f$  and  $g$  be the functions given by  $f(x) = 1 + \sin(2x)$  and  $g(x) = e^{x/2}$ . Let  $R$  be the shaded region in the first quadrant enclosed by the graphs of  $f$  and  $g$  as shown in the figure above.
- Find the area of  $R$ .
  - Find the volume of the solid generated when  $R$  is revolved about the  $x$ -axis.
  - The region  $R$  is the base of a solid. For this solid, the cross sections perpendicular to the  $x$ -axis are semicircles with diameters extending from  $y = f(x)$  to  $y = g(x)$ . Find the volume of this solid.
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**WRITE ALL WORK IN THE TEST BOOKLET.**

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2. A water tank at Camp Newton holds 1200 gallons of water at time  $t = 0$ . During the time interval  $0 \leq t \leq 18$  hours, water is pumped into the tank at the rate

$$W(t) = 95\sqrt{t} \sin^2\left(\frac{t}{6}\right) \text{ gallons per hour.}$$

During the same time interval, water is removed from the tank at the rate

$$R(t) = 275\sin^2\left(\frac{t}{3}\right) \text{ gallons per hour.}$$

- (a) Is the amount of water in the tank increasing at time  $t = 15$ ? Why or why not?
- (b) To the nearest whole number, how many gallons of water are in the tank at time  $t = 18$ ?
- (c) At what time  $t$ , for  $0 \leq t \leq 18$ , is the amount of water in the tank at an absolute minimum? Show the work that leads to your conclusion.
- (d) For  $t > 18$ , no water is pumped into the tank, but water continues to be removed at the rate  $R(t)$  until the tank becomes empty. Let  $k$  be the time at which the tank becomes empty. Write, but do not solve, an equation involving an integral expression that can be used to find the value of  $k$ .
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3. A particle moves along the  $x$ -axis so that its velocity  $v$  at time  $t$ , for  $0 \leq t \leq 5$ , is given by  $v(t) = \ln(t^2 - 3t + 3)$ . The particle is at position  $x = 8$  at time  $t = 0$ .
- (a) Find the acceleration of the particle at time  $t = 4$ .
  - (b) Find all times  $t$  in the open interval  $0 < t < 5$  at which the particle changes direction. During which time intervals, for  $0 \leq t \leq 5$ , does the particle travel to the left?
  - (c) Find the position of the particle at time  $t = 2$ .
  - (d) Find the average speed of the particle over the interval  $0 \leq t \leq 2$ .
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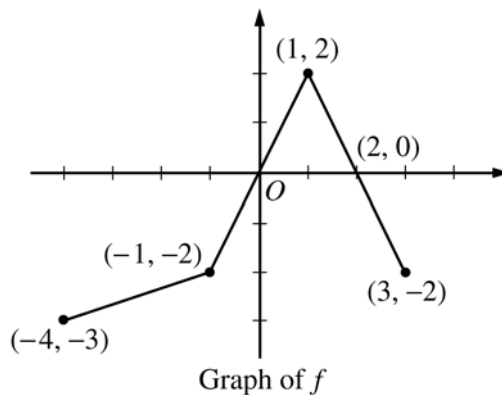
**WRITE ALL WORK IN THE TEST BOOKLET.**

**END OF PART A OF SECTION II**

**2005 AP<sup>®</sup> CALCULUS AB FREE-RESPONSE QUESTIONS (Form B)**

**CALCULUS AB**  
**SECTION II, Part B**  
**Time—45 minutes**  
**Number of problems—3**

No calculator is allowed for these problems.



4. The graph of the function  $f$  above consists of three line segments.
- (a) Let  $g$  be the function given by  $g(x) = \int_{-4}^x f(t) dt$ . For each of  $g(-1)$ ,  $g'(-1)$ , and  $g''(-1)$ , find the value or state that it does not exist.
- (b) For the function  $g$  defined in part (a), find the  $x$ -coordinate of each point of inflection of the graph of  $g$  on the open interval  $-4 < x < 3$ . Explain your reasoning.
- (c) Let  $h$  be the function given by  $h(x) = \int_x^3 f(t) dt$ . Find all values of  $x$  in the closed interval  $-4 \leq x \leq 3$  for which  $h(x) = 0$ .
- (d) For the function  $h$  defined in part (c), find all intervals on which  $h$  is decreasing. Explain your reasoning.

**WRITE ALL WORK IN THE TEST BOOKLET.**

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5. Consider the curve given by  $y^2 = 2 + xy$ .

(a) Show that  $\frac{dy}{dx} = \frac{y}{2y - x}$ .

(b) Find all points  $(x, y)$  on the curve where the line tangent to the curve has slope  $\frac{1}{2}$ .

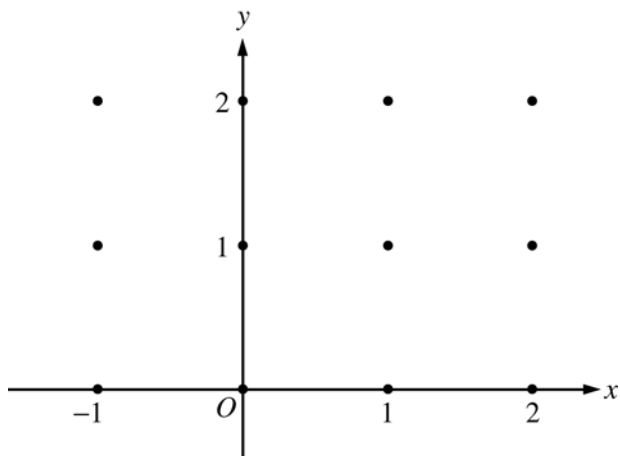
(c) Show that there are no points  $(x, y)$  on the curve where the line tangent to the curve is horizontal.

(d) Let  $x$  and  $y$  be functions of time  $t$  that are related by the equation  $y^2 = 2 + xy$ . At time  $t = 5$ , the value of  $y$  is 3 and  $\frac{dy}{dt} = 6$ . Find the value of  $\frac{dx}{dt}$  at time  $t = 5$ .

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6. Consider the differential equation  $\frac{dy}{dx} = \frac{-xy^2}{2}$ . Let  $y = f(x)$  be the particular solution to this differential equation with the initial condition  $f(-1) = 2$ .

(a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated. (Note: Use the axes provided in the test booklet.)



(b) Write an equation for the line tangent to the graph of  $f$  at  $x = -1$ .

(c) Find the solution  $y = f(x)$  to the given differential equation with the initial condition  $f(-1) = 2$ .

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**WRITE ALL WORK IN THE TEST BOOKLET.**

**END OF EXAM**