



AP[®] Calculus AB 2005 Free-Response Questions

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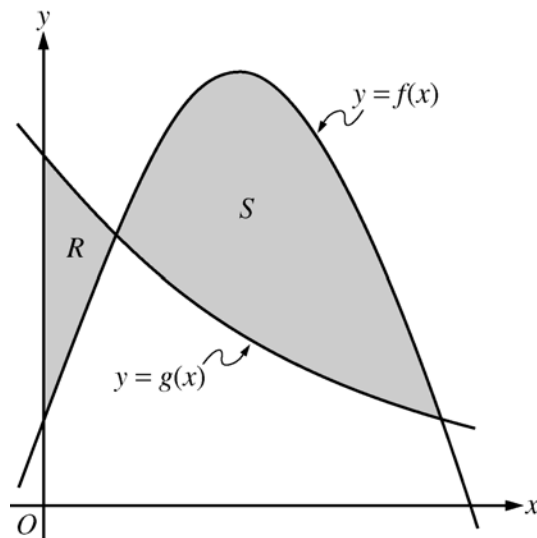
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2005 AP[®] CALCULUS AB FREE-RESPONSE QUESTIONS

CALCULUS AB
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.



1. Let f and g be the functions given by $f(x) = \frac{1}{4} + \sin(\pi x)$ and $g(x) = 4^{-x}$. Let R be the shaded region in the first quadrant enclosed by the y -axis and the graphs of f and g , and let S be the shaded region in the first quadrant enclosed by the graphs of f and g , as shown in the figure above.
- (a) Find the area of R .
- (b) Find the area of S .
- (c) Find the volume of the solid generated when S is revolved about the horizontal line $y = -1$.
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WRITE ALL WORK IN THE TEST BOOKLET.

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2. The tide removes sand from Sandy Point Beach at a rate modeled by the function R , given by

$$R(t) = 2 + 5 \sin\left(\frac{4\pi t}{25}\right).$$

A pumping station adds sand to the beach at a rate modeled by the function S , given by

$$S(t) = \frac{15t}{1 + 3t}.$$

Both $R(t)$ and $S(t)$ have units of cubic yards per hour and t is measured in hours for $0 \leq t \leq 6$. At time $t = 0$, the beach contains 2500 cubic yards of sand.

- How much sand will the tide remove from the beach during this 6-hour period? Indicate units of measure.
 - Write an expression for $Y(t)$, the total number of cubic yards of sand on the beach at time t .
 - Find the rate at which the total amount of sand on the beach is changing at time $t = 4$.
 - For $0 \leq t \leq 6$, at what time t is the amount of sand on the beach a minimum? What is the minimum value? Justify your answers.
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Distance x (cm)	0	1	5	6	8
Temperature $T(x)$ ($^{\circ}\text{C}$)	100	93	70	62	55

3. A metal wire of length 8 centimeters (cm) is heated at one end. The table above gives selected values of the temperature $T(x)$, in degrees Celsius ($^{\circ}\text{C}$), of the wire x cm from the heated end. The function T is decreasing and twice differentiable.
- (a) Estimate $T'(7)$. Show the work that leads to your answer. Indicate units of measure.
- (b) Write an integral expression in terms of $T(x)$ for the average temperature of the wire. Estimate the average temperature of the wire using a trapezoidal sum with the four subintervals indicated by the data in the table. Indicate units of measure.
- (c) Find $\int_0^8 T'(x) dx$, and indicate units of measure. Explain the meaning of $\int_0^8 T'(x) dx$ in terms of the temperature of the wire.
- (d) Are the data in the table consistent with the assertion that $T''(x) > 0$ for every x in the interval $0 < x < 8$? Explain your answer.
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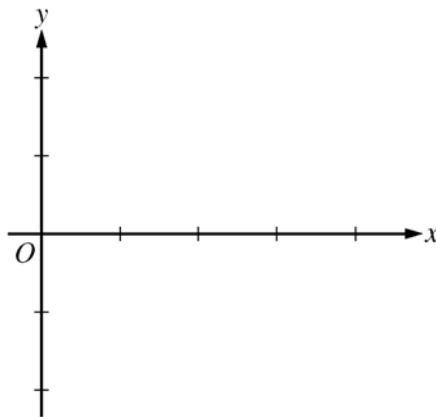
END OF PART A OF SECTION II

2005 AP[®] CALCULUS AB FREE-RESPONSE QUESTIONS**CALCULUS AB
SECTION II, Part B****Time—45 minutes****Number of problems—3****No calculator is allowed for these problems.**

x	0	$0 < x < 1$	1	$1 < x < 2$	2	$2 < x < 3$	3	$3 < x < 4$
$f(x)$	-1	Negative	0	Positive	2	Positive	0	Negative
$f'(x)$	4	Positive	0	Positive	DNE	Negative	-3	Negative
$f''(x)$	-2	Negative	0	Positive	DNE	Negative	0	Positive

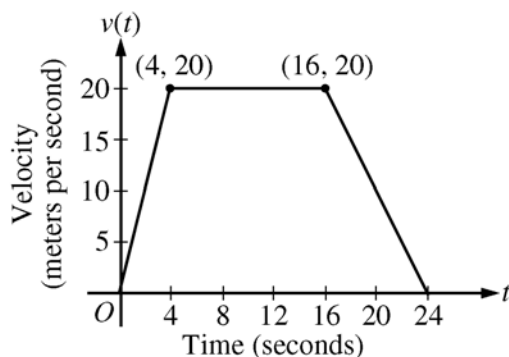
4. Let f be a function that is continuous on the interval $[0, 4)$. The function f is twice differentiable except at $x = 2$. The function f and its derivatives have the properties indicated in the table above, where DNE indicates that the derivatives of f do not exist at $x = 2$.
- (a) For $0 < x < 4$, find all values of x at which f has a relative extremum. Determine whether f has a relative maximum or a relative minimum at each of these values. Justify your answer.
- (b) On the axes provided, sketch the graph of a function that has all the characteristics of f .

(Note: Use the axes provided in the pink test booklet.)



- (c) Let g be the function defined by $g(x) = \int_1^x f(t) dt$ on the open interval $(0, 4)$. For $0 < x < 4$, find all values of x at which g has a relative extremum. Determine whether g has a relative maximum or a relative minimum at each of these values. Justify your answer.
- (d) For the function g defined in part (c), find all values of x , for $0 < x < 4$, at which the graph of g has a point of inflection. Justify your answer.

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5. A car is traveling on a straight road. For $0 \leq t \leq 24$ seconds, the car's velocity $v(t)$, in meters per second, is modeled by the piecewise-linear function defined by the graph above.
- (a) Find $\int_0^{24} v(t) dt$. Using correct units, explain the meaning of $\int_0^{24} v(t) dt$.
- (b) For each of $v'(4)$ and $v'(20)$, find the value or explain why it does not exist. Indicate units of measure.
- (c) Let $a(t)$ be the car's acceleration at time t , in meters per second per second. For $0 < t < 24$, write a piecewise-defined function for $a(t)$.
- (d) Find the average rate of change of v over the interval $8 \leq t \leq 20$. Does the Mean Value Theorem guarantee a value of c , for $8 < c < 20$, such that $v'(c)$ is equal to this average rate of change? Why or why not?
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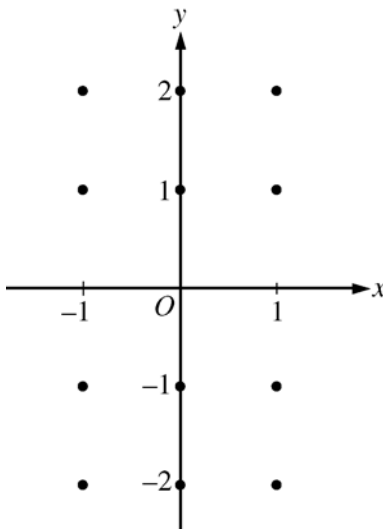
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6. Consider the differential equation $\frac{dy}{dx} = -\frac{2x}{y}$.

(a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated.

(Note: Use the axes provided in the pink test booklet.)



(b) Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(1) = -1$.

Write an equation for the line tangent to the graph of f at $(1, -1)$ and use it to approximate $f(1.1)$.

(c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition

$f(1) = -1$.

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END OF EXAM