CollegeBoard
Advanced Placement Program

AP® Calculus BC
2005 Scoring Commentary
Form B

Question 1

Overview

Students are given the rate of change of the x- and y-coordinates as functions of time and the initial position of a particle. Part (a) asks for the acceleration vector and speed at a specific time. Part (b) asks for the y-coordinate of the position at time $t = 2,$ requiring the evaluation of a definite integral of $\frac{dy}{dt}$ as an accumulator expressing change in the y-coordinate and adding it to the initial position. Part (c) tests whether students can find the slope of the line tangent to the graph of the particle's trajectory by asking for the equation of the line tangent to this curve at a specific point. Part (d) asks for the times at which the particle is at rest, testing whether students realize that both derivatives with respect to time must be zero.

Sample: 1A
Score: 9

The student earned all 9 points.

Sample: 1B
Score: 6

The student earned 3 points in part (b), 1 point in part (c), and 2 points in part (d). In part (a) the student makes a sign error in calculating the acceleration vector and does not find the speed. Part (b) is correct. In part (c) the student makes a calculation error in determining the slope but earned the equation point. Part (d) is correct.

Sample: 1C
Score: 4

The student earned 2 points in part (b) and 2 points in part (c). In part (a) the student has correct formulas but makes a differentiation error in determining acceleration and an arithmetic error in calculating speed. In part (b) the student earned two points for the setup but makes an incorrect evaluation so did not earn the answer point. In part (c) the student finds correct slope and imports the y-value obtained in part (b) and earned both points. The student did not earn any points in part (d).

Question 2

Overview

The problem presents a situation in which water is being pumped into, and removed from, a tank. Functions modeling the rates at which water is entering and leaving the tank are given. Part (a) tests students' ability to combine these rates by asking whether or not the amount of water in the tank is increasing at a specific time and the reason for this. Part (b) tests knowledge of the Fundamental Theorem of Calculus. It requires knowing how to use the rates and the initial amount of water in the tank to determine the amount of water in the tank at a given time. Part (c) asks for the time at which the water is at an absolute minimum. Students can check the water level at the endpoints and at the times when the rates are equal, or they can argue that since the rate of removal is greater than the rate at which water was pumped in over two intervals, one at the beginning and one at the end of the 18-hour period, the minimum can only occur at the first time $t > 0$ at which these curves cross or at time $t = 18$. After making this argument, they can then check the water level at those two times. Part (d) requires setting up an equation in which the integral of the rate of removal from $t = 18$ to $t = k$ is equated to the amount of water the student states is in the tank at time $t = 18$.

Sample: 2A
Score: 9

The student earned all 9 points.

Sample: 2B
Score: 6

The student earned 3 points in part (b), 1 point in part (c), and 2 points in part (d). In part (a) the information from the student-created graph did not earn any points. In part (b) the student correctly figures the number of gallons at time $t = 18$. Although the answer 1309.79 does not meet the requirement of three decimal places, the student circles the final answer, 1310 gallons, which is correct, rounded to the nearest whole number as prompted in the question. In part (c) the student did not earn the first point for considering only one interior critical point. The second point was earned for naming the time at which the minimum occurs. The student does not show evidence of the absolute minimum and did not earn the third point. In part (d) the student writes the correct limits and sets up the equation correctly, earning both points.

Sample: 2B
Score: 4

The student earned 1 point in part (a), 2 points in part (b), and 1 point in part (d). Part (a) is correct. In part (b) the student earned the points for correct limits and correct integrand but did not earn the answer point, because the initial amount of 1200 gallons is not added. In part (c) the student does not address the question in any way. In part (d) the student earned the point for correct limits. The second point was not earned because the integral is equated to 0.

Question 3

Overview

Students are given an expression for the nth derivative of f at $x = 0$. In part (a) they are asked to determine with justification whether f has an extremum and, if so, what type, at $x = 0$. In part (b) students are tested on whether they can use the derivatives to find the third-degree polynomial approximation to f about $x = 0$. In part (c) students are asked to determine the radius of convergence. This can be done using the Ratio Test.

Sample: 3A
Score: 9

The student earned all 9 points. In part (c) the student earned two points for the second line, one for the general term, and one for the ratio. The limit is shown in the third line and the radius is found, which earned the final two points.

Sample: 3B
Score: 6

The student earned 1 point in part (a), 1 point in part (b), and 4 points in part (c). In part (a) the student notes that the first derivative is 0 and the second derivative is negative at $x = 0$. However, the student fails to commit to an answer, so the answer point was not earned. In part (b) the student has two incorrect terms, which resulted in only 1 point being earned. In part (c) the student earned points for the general term and for setting up the ratio. In applying the limit process in the next line, the student concludes that the radius of convergence is 5.

Sample: 3C
Score: 3

The student earned 1 point in part (b) and 2 points in part (c). In part (a) the student incorrectly concludes that the first and second derivatives are not defined. In part (b) the student finds only two correct terms and was eligible to earn two points. However, the student lost a point for concluding that $f(x)$ equals the Taylor polynomial, which is not true for this function. Thus the student earned 1 point in part (b). In part (c) the student earned the first 2 points for the general term and for the ratio of terms.

Question 4

Overview

Students are given the graph of a piecewise-linear function f and asked questions about two functions: $g(x) = \int_{-4}^{x} f(t)\,dt$ and $h(x) = \int_{x}^{3} f(t)\,dt$. Part (a) asks for values of g and its first and second derivatives at $x = -1$. This tests students' ability to equate the derivative of g with f and to read information about g from the graph of f. Part (b) asks for the inflection point of the graph of g, testing whether students can identify it as the point at which f changes from increasing to decreasing, regardless of the fact that the derivative of f, and thus the second derivative of g, does not exist at this point. Part (c) asks for the values of x at which h is zero, testing students' ability to interpret the integral as an area. In part (d) students are asked to find the intervals on which h is decreasing. Students should know that the derivative of h is the negative of f, but they also can answer this question by considering how h changes as a function of x.

Sample: 4A
Score: 9

The student earned all 9 points. The reason point in part (b) was earned for making the connection between g'' and f' and stating that f' changes sign. In part (d) the student earned the point for the open interval $0 < x < 2$ and the reason point for the correct connection of h'' to $-f$. The use of t instead of x is ignored.

Sample: 4B
Score: 6

The student earned 3 points in part (a), 1 point in part (c), and 2 points in part (d). Part (a) is correct. The student did not earn any points in part (b). In part (c) the student earned one of the 2 points for giving two of the three correct values of x. The student earned both points in part (d).

Sample: 4C
Score: 3

The student earned 2 points in part (a) and 1 point in part (b). The student did not earn any points in parts (c) and (d). In part (a) the student earned the points for $g'(-1)$ and $g''(-1)$ but did not earn the point for $g(-1)$ because of the incorrect sign. In part (c) the student is missing two of the three correct values of x and did not earn any points.

Question 5

Overview

Students are given an implicitly defined curve and asked to verify that the derivative is a specific function of x and y. This tests implicit differentiation. The solution is given so that students can work parts (b) and (c) without having to be successful in part (a). Part (b) tests whether students can combine the information in the equation of the curve with the formula for the derivative to find where the graph of the function has slope $\frac{1}{2}$. Part (c) pushes this a little further. Students have to observe that if the derivative is 0, then $y = 0$, but that there is no point with y-coordinate 0 on the curve. Part (d) requires students to use the chain rule to find $\frac{dx}{dt}$ using their knowledge of $\frac{dy}{dx}$ and $\frac{dy}{dt}$.

Sample: 5A
Score: 9

The student earned all 9 points. In part (c) the student earned the $y = 0$ point because the numerator of the derivative is equated to 0. The explanation point was earned for explaining that substituting $y = 0$ in the curve results in a false statement. In part (d) the student earned the solves for x point by identifying $x = \frac{7}{3}$ when $y = 3$, the chain rule point for correctly differentiating the curve with respect to time, and the answer point for correct evaluation.

Sample: 5B
Score: 6

The student earned 2 points in part (a), 2 points in part (b), 1 point in part (c), and 1 point in part (d). Parts (a) and (b) are correct. In part (c) the student earned the $y = 0$ point. The student does not explain why there are no points on the curve where the tangent to the curve is horizontal. As a result, the student did not earn the explanation point. In part (d) the student earned the chain rule point for correctly differentiating the curve with respect to time. However, the student never solves for x and instead uses the value of $t = 5$ for x. The student was not eligible for the answer point.

Sample: 5C
Score: 4

The student earned 2 points in part (a), 1 point in part (b), and 1 point in part (d). Part (a) is correct. In part (b) the student earned the point for setting the derivative equal to $\frac{1}{2}$. Since only one of the points where the derivative is equal to $\frac{1}{2}$ is found, the student did not earn the answer point. The student did not earn any points in part (c). The student does not state that $y = 0$ and gives no explanation. In part (d) the student earned the implicit differentiation point. The student does not solve for x or give an answer.

Question 6

Overview

Parts (a) and (b) asks for an area and a volume of a solid of revolution. What distinguishes these problems is that the answer is in terms of a parameter k, the x-coordinate of the right-hand boundary of the region. For part (c), students have to set up an improper integral representing the volume of a solid of revolution of a region bounded on the left by $x = k$ and unbounded on the right. Students have to set this integral equal to the volume found in part (b), evaluate the improper integral, and solve for k.

Sample: 6A
Score: 9

The student earned all 9 points. In part (c) the student presents the correct steps for handling the improper integral.

Sample: 6B
Score: 6

The student earned 2 points in part (a), 2 points in part (b), and 2 points in part (c). Part (a) is correct. In part (b) the student earned 2 points for the correct volume integral expression. In part (c) the student correctly writes the volume integral for region S and equates it to the volume for region R from part (b), which earned 2 points. The student does not complete the problem.

Sample: 6C
Score: 3

The student earned 1 point in part (a) and 2 points in part (b). In part (a) the student earned 1 point for the proper integral for the area of region R but does not complete the problem. In part (b) the student earned two points for writing the correct integral for the volume of region R but does not complete the problem. The student did not earn any points in part (c).