CollegeBoard
Advanced Placement Program

AP® Calculus AB
2005 Scoring Commentary

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Question 1

Overview

This problem gives two functions whose graphs intersect at $x \approx 0.178218$ and at $x = 1$. Students have to find these first two points of intersection to the right of the y-axis. In part (a) students have to find the area of the region bounded by the y-axis and the two graphs. In part (b) they have to find the area bounded by the two graphs between the first two points of intersection. Part (c) tests whether students can set up and then evaluate the definite integral that represents the volume of the solid of revolution obtained when the region, whose area is found in part (b), is revolved about the horizontal line $y = -1$.

Sample: 1A
Score: 9

The student earned all 9 points.

Sample: 1B
Score: 6

The student earned 3 points in part (a), 1 point in part (b), and 2 points in part (c). The work in part (a) is correct. In part (b) the upper limit in the integral is incorrect, so the limits point was not earned. The integrand in part (b) is correct, but the student was not eligible for the answer point because the limits point was not earned. In part (c) the student has a correct integrand and earned two points. Even though the limits of integration are consistent with those in part (b), the student omits the constant π and did not earn the answer point.

Sample: 1C
Score: 4

The student earned 1 point in part (a) and 3 points in part (b). In part (a) the student earned only the integrand point for a correct integrand. The student has incorrect limits of integration; the upper limit is the y-coordinate of the point of intersection rather than the x-coordinate. The student was already penalized for using 0.781 as the x-coordinate of the first point of intersection and so is allowed to use this value in part (b). The answer to part (b) is consistent with the student's limits, so the student earned all 3 points. In part (c) the student's integrand does not reflect rotation about the horizontal line $y = -1$; it is set up as if the region were being rotated about the x-axis. This simplifies the problem so the student could not earn the two integrand points. Since the integrand points were not earned, it was not possible for the student to earn the answer point. Note also the incorrect limits of integration and the negative answer for the volume.

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Question 2

Overview

This problem describes a situation in which sand is simultaneously being removed from a beach by the tide and replaced by a pumping station. Functions that model the rates of removal and replacement are provided, and students are given the initial amount of sand. Parts (a) and (b) test knowledge of the Fundamental Theorem of Calculus by challenging students to find the total amount of sand removed by the tide over the first six hours and to find an expression, necessarily an integral expression or one obtained through integration, for the total amount of sand on the beach at time t. Part (c) tests whether students can combine the rates correctly to find the rate at which the total amount of sand on the beach is changing. Part (d) tests whether students can then use this combined rate to find the time at which the amount of sand is at its minimum, justify this answer, and use the Fundamental Theorem of Calculus or the solution to part (b) to give the amount of sand at that time.

Sample: 2A
Score: 9

The student earned all 9 points. In part (d) the student successfully performs a candidate test for a global minimum, identifying the critical point and comparing function values at the critical point and the endpoints of the interval.

Sample: 2B
Score: 6

The student earned 2 points in part (a), 1 point in part (b), 1 point in part (c), and 2 points in part (d). Part (a) is correct, and the student earned both points. In part (b) the student earned the integrand point but lost the limit point and answer point since the integral is left as an indefinite integral, and the initial condition is never considered. The work in part (c) is correct, and the student earned the point. In part (d) the student earned 1 point for setting the derivative of $Y(t)$ equal to 0 and 1 point for correctly identifying the critical value for t. The student did not earn the point for answer with justification. No answer for the minimum amount of sand is given, and the argument for a minimum is a local one.

Sample: 2C
Score: 3

The student earned 1 point in part (a), 1 point in part (b), and 1 point in part (c). In part (a) the student earned the integral point but did not earn the second point since the correct numerical answer has incorrect units. The student reports the units as yards instead of cubic yards. In part (b) the student did not earn the integrand point and answer point. The integrand given is $R(t) + S(t)$ instead of the correct $S(t) - R(t)$. The student also does not address the initial condition. The student does have the correct limits of integration, which earned the limits point. In part (c) the student works with $-R(4) + S(4)$ and gets the correct answer, which earned the point. No points were earned in part (d), since the student does not set the derivative of $Y(t)$ equal to 0 and does not identify the critical value of t. The student also does not determine the minimum amount of sand.

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Question 3

Overview

This problem presents students with a table of the temperatures, $T(x)$, of a wire taken at intervals of unequal lengths, where x is the distance from the heated end of the wire. Part (a) asks for an estimation of the derivative of T at a point midway between two of the table values. It is expected that students will use the average rate of change of T over this interval to approximate the derivative. Part (b) requires setting up an integral for the average temperature and using all of the information in the table to find a trapezoidal approximation of the integral over the length of the wire. Stating correct units for average temperature is an important part of this problem. Part (c) tests student knowledge of the Fundamental Theorem of Calculus. Students have to recognize $\int_0^8 T'(x)\,dx$ as the total change in the value of T between the endpoints at $x = 0$ and at $x = 8$. Part (d) asks whether the tabular data are consistent with a strictly positive second derivative at each point between $x = 0$ and $x = 8$. One way to do this is to find average rates of change over successive intervals of unequal lengths and to recognize that since these average rates of change are not increasing, the second derivative cannot be positive on the entire interval. Ideally, students should invoke the Mean Value Theorem to make the connection between the average rate of change and the value of the first derivative at some point in the interval.

Sample: 3A
Score: 9

The student earned all 9 points.

Sample: 3B
Score: 6

The student earned 1 point in part (a), 1 point in part (b), 2 points in part (c), 1 point in part (d), and the units point. In part (b) the student earned the first point for correctly setting up the integral. No points were awarded for the trapezoidal sum or answer, since the student assumes intervals of equal length instead of using the data from the table. In part (d) the first point was earned for two correct slopes of secant lines. The second point was not earned. The student skips over the interval from 5 to 6, which shows that T'' decreased.

Sample: 3C
Score: 4

The student earned 1 point in part (a), 1 point in part (b), 1 point in part (c), and 1 point in part (d). The student did not earn the units point since part (c) includes $^\circ\text{C/cm}$. In part (b) the student earned the first point but does not use the data to set up intervals of the correct lengths for the trapezoidal approximation and thus did not earn the setup or answer points. In part (c) the student earned the first point but did not receive credit for the incorrect explanation involving average rate of change. In part (d) the student earned the first point with "decreases 7" and "-5.75," but since there is an incorrect slope of -10, the answer point was not earned.

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Question 4

Overview

In this problem, students are given the values of a continuous function f and its first and second derivatives at four points, together with the signs of these functions in the unit interval to the right of each of these four points. Part (a) tests whether students can use the information in the table to identify the locations of the relative extrema of f and justify their answers. Part (b) asks for a sketch of the function. Students need to make the critical observation that if a continuous function is increasing and its graph is concave up to the left of $x = 2$ and decreasing and its graph is concave down to the right of $x = 2,$ then there must be a point of nondifferentiability at $x = 2.$ Parts (c) and (d) test knowledge of the Fundamental Theorem of Calculus, defining g as the definite integral of f from $t = 1$ to $t = x.$ Students are asked to find and justify the relative extrema of g and the points of inflection of the graph of g.

Sample: 4A
Score: 9

The student earned all 9 points. In part (a) the argument concerning $x = 0$ is ignored since this lies outside the interval given in the problem.

Sample: 4B
Score: 6

The student earned 1 point in part (a), 2 points in part (b), 2 points in part (c), and 1 point in part (d). In part (a) the student correctly identifies $x = 2$ as the relative maximum. Although the student is correct in stating that f is increasing to the left of 2 and decreasing to the right of 2, this is considered to be the rephrasing of the definition of a relative maximum and is not a sufficient argument. The sign chart is not adequate to complete the justification. The graph is correct. In part (c) the student notes that $g' = f$ and finds the correct critical points. The justification is similar to that in part (a) and is not adequate. In part (d) the student correctly identifies the point of inflection, but the justification is not adequate.

Sample: 4C
Score: 3

The student earned 1 point in part (b) and 2 points in part (c). In part (a) the student has the incorrect x-value for the extremum. In part (b) the graph passes through the points and is not differentiable at $x = 2,$ but the concavity is incorrect. In part (c) the student identifies the critical points and states that $g' = f$ but discusses the sign behavior of f' rather than that of $f.$ In part (d) the student fails to identify the point of inflection.

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Question 5

Overview

Students are given a piecewise-linear graph that models a car's velocity at time t. Testing the ability to connect integrals with area under a graph, part (a) asks students to find the integral of v from $t = 0$ to $t = 24$ and to explain the meaning of this integral, that is to say that it represents the change in position over that time interval. Part (b) asks students to determine values of the derivative of v from the graph and to indicate units of measure. Part (c) asks for a piecewise-defined function that describes the car's acceleration. Part (d) requires students to calculate the average rate of change of v and whether the Mean Value Theorem would guarantee the existence of a point on the interval where the derivative took on that value. In parts (b) and (c) students are led to discover points at which the derivative of v is not defined. This question tests whether students recognize that the Mean Value Theorem does not apply when the derivative of v is not defined over the entire interval.

Sample: 5A
Score: 9

The student earned all 9 points.

Sample: 5B
Score: 6

The student earned 2 points in part (a), 2 points in part (b), 1 point in part (c), and 1 point in part (d). In part (b) the student does not exhibit correct units of $v'(t)$ and did not earn the third point. In part (c) the student earned the first point for the values of $a(t)$ but not the second point, due to the incorrect treatment at $t = 4$ and at $t = 16.$ In part (d) the student earned the second point by stating which hypothesis of the Mean Value Theorem is not satisfied. Endpoint behavior is not considered relevant unless the student specifically invokes it in the justification.

Sample: 5C
Score: 4

The student earned 1 point in part (a), 1 point in part (b), 1 point in part (c), and 1 point in part (d). In part (a) there is no reference to meters, and the student did not earn the second point. In part (b) the student did not earn the point for $v'(20)$. In part (c) the student earned the first point for the values of $a(t)$ but not the second point, due to the incorrect treatment at $t = 4$ and at $t = 16.$ In part (d) units are not necessary since they are not requested in the problem. The student did not earn the second point since the statement regarding the Mean Value Theorem is incorrect.

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Question 6

Overview

Students are presented with a separable differential equation and asked to sketch its slope field in part (a). In part (b) students have to know how to use the differential equation to find the equation of the line tangent to the solution curve through the point $(1, -1)$. They are also asked to use this tangent to approximate the value of the particular solution at $x = 1.1$. Part (c) requires solving the separable differential equation to find an exact formula for the solution with $f(1) = -1$. There are two branches to the solution, and students have to choose the correct branch.

Sample: 6A
Score: 9

The student earned all 9 points. In part (c) the student earned the constant of integration point in line 4 because it is correctly used.

Sample: 6B
Score: 6

The student earned 2 points in part (a), 2 points in part (b), and 2 points in part (c). In part (a) slopes are zero on the y-axis, and nonzero slopes are drawn correctly. In part (b) the student earned the first point in the first line with the equation $y + 1 = 2(x - 1)$; the approximation for $f(1.1)$ is also correct. In part (c) the student correctly separated the variables (for the first point) and found both antiderivatives correctly (for the second point). Since no constant of integration is present, no further points could be earned.

Sample: 6C
Score: 4

The student earned 2 points in part (a), 1 point in part (b), and 1 point in part (c). All slope lines are drawn correctly in part (a) for 2 points. In part (b) the equation of the tangent line $y + 1 = 2(x - 1)$ earned the first point; however, due to an error in simplifying the equation, the student did not earn the approximation point. In part (c) the student earned 1 point for the correct separation of variables.

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