AP® Physics C: Mechanics
2012 Free-Response Questions

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TABLE OF INFORMATION DEVELOPED FOR 2012

CONSTANTS AND CONVERSION FACTORS			
Proton mass,	$m_p = 1.67 \times 10^{-27}$ kg	Electron charge magnitude,	$e = 1.60 \times 10^{-19}$ C
Neutron mass,	$m_n = 1.67 \times 10^{-27}$ kg	1 electron volt,	$1\text{ eV} = 1.60 \times 10^{-19}$ J
Electron mass,	$m_e = 9.11 \times 10^{-31}$ kg	Speed of light,	$c = 3.00 \times 10^{8}$ m/s
Avogadro's number,	$N_0 = 6.02 \times 10^{23}$ mol^{-1}	Universal gravitational constant,	$G = 6.67 \times 10^{-11}$ m^3/kg•s^2
Universal gas constant,	$R = 8.31$ J/(mol•K)	Acceleration due to gravity at Earth's surface,	$g = 9.8$ m/s^2
Boltzmann's constant,	$k_B = 1.38 \times 10^{-23}$ J/K		

1 unified atomic mass unit,	$1\text{ u} = 1.66 \times 10^{-27}\text{ kg} = 931\text{ MeV}/c^2$
Planck's constant,	$h = 6.63 \times 10^{-34}\text{ J•s} = 4.14 \times 10^{-15}\text{ eV•s}$
	$hc = 1.99 \times 10^{-25}\text{ J•m} = 1.24 \times 10^{3}\text{ eV•nm}$
Vacuum permittivity,	$\epsilon_0 = 8.85 \times 10^{-12}\text{ C}^2/\text{N•m}^2$
Coulomb's law constant,	$k = 1/4\pi\epsilon_0 = 9.0 \times 10^{9}\text{ N•m}^2/\text{C}^2$
Vacuum permeability,	$\mu_0 = 4\pi \times 10^{-7}\text{ (T•m)/A}$
Magnetic constant,	$k' = \mu_0/4\pi = 1 \times 10^{-7}\text{ (T•m)/A}$
1 atmosphere pressure,	$1\text{ atm} = 1.0 \times 10^{5}\text{ N/m}^2 = 1.0 \times 10^{5}\text{ Pa}$

UNIT SYMBOLS								
	meter,	m	mole,	mol	watt,	W	farad,	F
	kilogram,	kg	hertz,	Hz	coulomb,	C	tesla,	T
	second,	s	newton,	N	volt,	V	degree Celsius,	°C
	ampere,	A	pascal,	Pa	ohm,	Ω	electron-volt,	eV
	kelvin,	K	joule,	J	henry,	H		

PREFIXES		
Factor	Prefix	Symbol
10^{9}	giga	G
10^{6}	mega	M
10^{3}	kilo	k
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p

VALUES OF TRIGONOMETRIC FUNCTIONS FOR COMMON ANGLES							
θ	0°	30°	37°	45°	53°	60°	90°
$\sin\theta$	0	$1/2$	$3/5$	$\sqrt{2}/2$	$4/5$	$\sqrt{3}/2$	1
$\cos\theta$	1	$\sqrt{3}/2$	$4/5$	$\sqrt{2}/2$	$3/5$	$1/2$	0
$\tan\theta$	0	$\sqrt{3}/3$	$3/4$	1	$4/3$	$\sqrt{3}$	∞

The following conventions are used in this exam.

I. Unless otherwise stated, the frame of reference of any problem is assumed to be inertial.

II. The direction of any electric current is the direction of flow of positive charge (conventional current).

III. For any isolated electric charge, the electric potential is defined as zero at an infinite distance from the charge.

MECHANICS

$$v = v_0 + at$$

$$x = x_0 + v_0 t + \frac{1}{2}at^2$$

$$v^2 = v_0^2 + 2a(x - x_0)$$

$$\sum \mathbf{F} = \mathbf{F}_{net} = m\mathbf{a}$$

$$\mathbf{F} = \frac{d\mathbf{p}}{dt}$$

$$\mathbf{J} = \int \mathbf{F}\,dt = \Delta \mathbf{p}$$

$$\mathbf{p} = m\mathbf{v}$$

$$F_{fric} \le \mu N$$

$$W = \int \mathbf{F} \cdot d\mathbf{r}$$

$$K = \frac{1}{2}mv^2$$

$$P = \frac{dW}{dt}$$

$$P = \mathbf{F} \cdot \mathbf{v}$$

$$\Delta U_g = mgh$$

$$a_c = \frac{v^2}{r} = \omega^2 r$$

$$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$$

$$\sum \boldsymbol{\tau} = \boldsymbol{\tau}_{net} = I\boldsymbol{\alpha}$$

$$I = \int r^2 dm = \sum mr^2$$

$$\mathbf{r}_{cm} = \sum m\mathbf{r} / \sum m$$

$$v = r\omega$$

$$\mathbf{L} = \mathbf{r} \times \mathbf{p} = I\boldsymbol{\omega}$$

$$K = \frac{1}{2}I\omega^2$$

$$\omega = \omega_0 + \alpha t$$

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$

a = acceleration
F = force
f = frequency
h = height
I = rotational inertia
J = impulse
K = kinetic energy
k = spring constant
ℓ = length
L = angular momentum
m = mass
N = normal force
P = power
p = momentum
r = radius or distance
$\mathbf{r}$ = position vector
T = period
t = time
U = potential energy
v = velocity or speed
W = work done on a system
x = position
μ = coefficient of friction
θ = angle
τ = torque
ω = angular speed
α = angular acceleration
ϕ = phase angle

$$\mathbf{F}_s = -k\mathbf{x}$$

$$U_s = \frac{1}{2}kx^2$$

$$x = x_{\max}\cos(\omega t + \phi)$$

$$T = \frac{2\pi}{\omega} = \frac{1}{f}$$

$$T_s = 2\pi\sqrt{\frac{m}{k}}$$

$$T_p = 2\pi\sqrt{\frac{\ell}{g}}$$

$$\mathbf{F}_G = -\frac{Gm_1m_2}{r^2}\hat{\mathbf{r}}$$

$$U_G = -\frac{Gm_1m_2}{r}$$

ELECTRICITY AND MAGNETISM

$$F = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r^2}$$

$$\mathbf{E} = \frac{\mathbf{F}}{q}$$

$$\oint \mathbf{E} \cdot d\mathbf{A} = \frac{Q}{\epsilon_0}$$

$$E = -\frac{dV}{dr}$$

$$V = \frac{1}{4\pi\epsilon_0}\sum_i \frac{q_i}{r_i}$$

$$U_E = qV = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r}$$

$$C = \frac{Q}{V}$$

$$C = \frac{\kappa\epsilon_0 A}{d}$$

$$C_p = \sum_i C_i$$

$$\frac{1}{C_s} = \sum_i \frac{1}{C_i}$$

$$I = \frac{dQ}{dt}$$

$$U_c = \frac{1}{2}QV = \frac{1}{2}CV^2$$

$$R = \frac{\rho\ell}{A}$$

$$\mathbf{E} = \rho\mathbf{J}$$

$$I = Nev_d A$$

$$V = IR$$

$$R_s = \sum_i R_i$$

$$\frac{1}{R_p} = \sum_i \frac{1}{R_i}$$

$$P = IV$$

$$\mathbf{F}_M = q\mathbf{v} \times \mathbf{B}$$

A = area
B = magnetic field
C = capacitance
d = distance
E = electric field
$\mathcal{E}$ = emf
F = force
I = current
J = current density
L = inductance
ℓ = length
n = number of loops of wire per unit length
N = number of charge carriers per unit volume
P = power
Q = charge
q = point charge
R = resistance
r = distance
t = time
U = potential or stored energy
V = electric potential
v = velocity or speed
ρ = resistivity
ϕ_m = magnetic flux
κ = dielectric constant

$$\oint \mathbf{B} \cdot d\boldsymbol{\ell} = \mu_0 I$$

$$d\mathbf{B} = \frac{\mu_0}{4\pi}\frac{I\,d\boldsymbol{\ell} \times \mathbf{r}}{r^3}$$

$$\mathbf{F} = \int I\,d\boldsymbol{\ell} \times \mathbf{B}$$

$$B_s = \mu_0 nI$$

$$\phi_m = \int \mathbf{B} \cdot d\mathbf{A}$$

$$\mathcal{E} = \oint \mathbf{E} \cdot d\boldsymbol{\ell} = -\frac{d\phi_m}{dt}$$

$$\mathcal{E} = -L\frac{dI}{dt}$$

$$U_L = \frac{1}{2}LI^2$$

GEOMETRY AND TRIGONOMETRY

Rectangle

$A = bh$

Triangle

$A = \frac{1}{2}bh$

Circle

$A = \pi r^2$

$C = 2\pi r$

Rectangular Solid

$V = \ell wh$

Cylinder

$V = \pi r^2 \ell$

$S = 2\pi r\ell + 2\pi r^2$

Sphere

$V = \frac{4}{3}\pi r^3$

$S = 4\pi r^2$

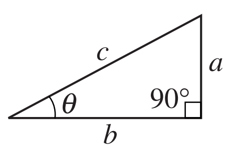
Right Triangle

$a^2 + b^2 = c^2$

$\sin\theta = \frac{a}{c}$

$\cos\theta = \frac{b}{c}$

$\tan\theta = \frac{a}{b}$



A = area
C = circumference
V = volume
S = surface area
b = base
h = height
ℓ = length
w = width
r = radius

CALCULUS

$$\frac{df}{dx} = \frac{df}{du}\frac{du}{dx}$$

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$$

$$\frac{d}{dx}\left(e^x\right) = e^x$$

$$\frac{d}{dx}(\ln x) = \frac{1}{x}$$

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1},\ n \neq -1$$

$$\int e^x\,dx = e^x$$

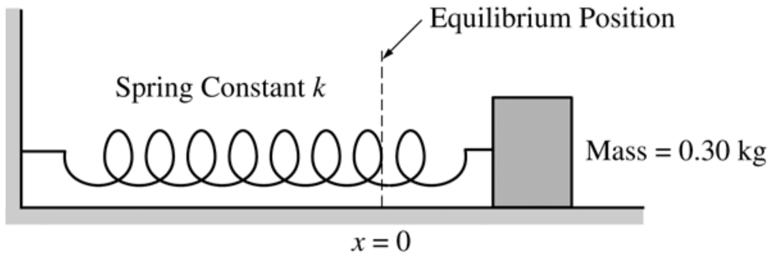
$$\int \frac{dx}{x} = \ln|x|$$

$$\int \cos x\,dx = \sin x$$

$$\int \sin x\,dx = -\cos x$$

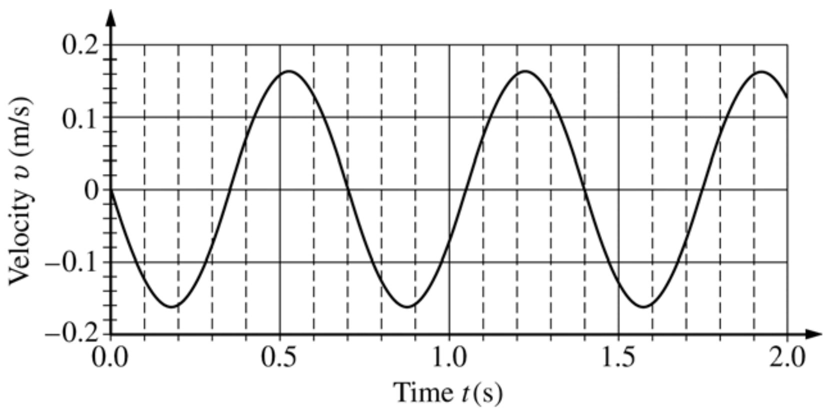
PHYSICS C: MECHANICS
SECTION II
Time—45 minutes
3 Questions

Directions: Answer all three questions. The suggested time is about 15 minutes for answering each of the questions, which are worth 15 points each. The parts within a question may not have equal weight. Show all your work in this booklet in the spaces provided after each part.



Mech. 1.

Experiment 1. A block of mass 0.30 kg is placed on a frictionless table and is attached to one end of a horizontal spring of spring constant k, as shown above. The other end of the spring is attached to a fixed wall. The block is set into oscillatory motion by stretching the spring and releasing the block from rest at time $t = 0$. A motion detector is used to record the position of the block as it oscillates. The resulting graph of velocity v versus time t is shown below. The positive direction for all quantities is to the right.



(a) Determine the equation for $v(t)$, including numerical values for all constants.

(b) Given that the equilibrium position is at $x = 0$, determine the equation for $x(t)$, including numerical values for all constants.

(c) Calculate the value of k.

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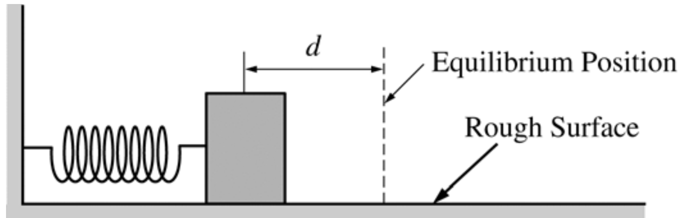
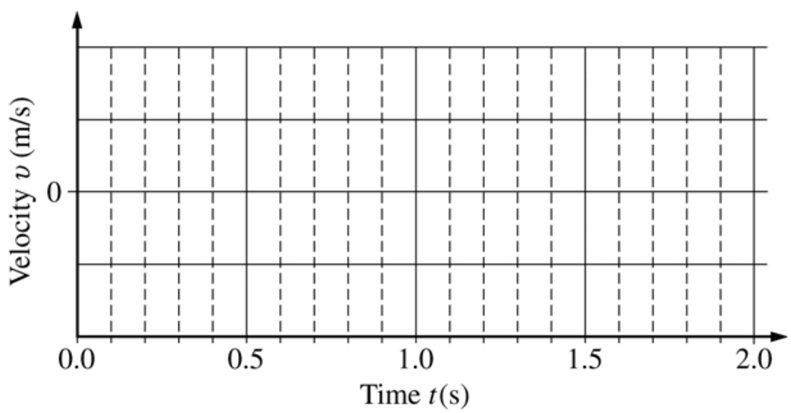
Experiment 2. The block and spring arrangement is now placed on a rough surface, as shown below. The block is displaced so that the spring is compressed a distance d and released from rest.



(d) On the dots below that represent the block, draw and label the forces (not components) that act on the block when the spring is compressed a distance $x = d/2$ and the block is moving in the direction indicated below each dot.

Toward the equilibrium position	Away from the equilibrium position

(e) Draw a sketch of v versus t in this case. Assume that there is a negligible change in the period and that the positive direction is still to the right.



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Mech. 2.

You are to perform an experiment investigating the conservation of mechanical energy involving a transformation from initial gravitational potential energy to translational kinetic energy.

(a) You are given the equipment listed below, all the supports required to hold the equipment, and a lab table. On the list below, indicate each piece of equipment you would use by checking the line next to each item.

____ Track	____ Meterstick	____ Set of objects of different masses
____ Cart	____ Electronic balance	____ Lightweight low-friction pulley
____ String	____ Stopwatch	

(b) Outline a procedure for performing the experiment. Include a diagram of your experimental setup. Label the equipment in your diagram. Also include a description of the measurements you would make and a symbol for each measurement.

(c) Give a detailed account of the calculations of gravitational potential energy and translational kinetic energy both before and after the transformation, in terms of the quantities measured in part (b).

(d) After your first trial, your calculations show that the energy <u>increased</u> during the experiment. Assuming you made no mathematical errors, give a reasonable explanation for this result.

(e) On all other trials, your calculations show that the energy <u>decreased</u> during the experiment. Assuming you made no mathematical errors, give a reasonable physical explanation for the fact that the average energy you determined decreased. Include references to conservative and nonconservative forces, as appropriate.

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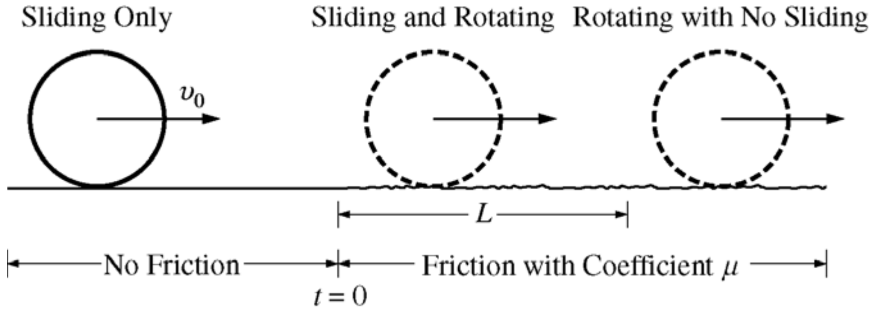

Mech. 3.

A ring of mass M, radius R, and rotational inertia MR^2 is initially sliding on a frictionless surface at constant velocity v_0 to the right, as shown above. At time $t = 0$ it encounters a surface with coefficient of friction μ and begins sliding and rotating. After traveling a distance L, the ring begins rolling without sliding. Express all answers to the following in terms of M, R, v_0, μ, and fundamental constants, as appropriate.

(a) Starting from Newton's second law in either translational or rotational form, as appropriate, derive a differential equation that can be used to solve for the magnitude of the following as the ring is sliding and rotating.

i. The linear velocity v of the ring as a function of time t

ii. The angular velocity ω of the ring as a function of time t

(b) Derive an expression for the magnitude of the following as the ring is sliding and rotating.

i. The linear velocity v of the ring as a function of time t

ii. The angular velocity ω of the ring as a function of time t

(c) Derive an expression for the time it takes the ring to travel the distance L.

(d) Derive an expression for the magnitude of the velocity of the ring immediately after it has traveled the distance L.

(e) Derive an expression for the distance L.

STOP

END OF EXAM

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