CALCULUS AB
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.

Work for problem 1(a)

\[ f(y) = \frac{1}{4} + \sin(\pi x) \]
\[ g(x) = 4 - x \]

\[ A_R = \int_0^{0.178} g(x) - f(x) \, dx \]

\[ A_R = \int_0^{0.178} 4 - x - \frac{1}{4} - \sin(\pi x) \, dx \]

\[ A_R = 0.0648 \text{ u}^2 \]

Continue problem 1 on page 5.
Work for problem 1(b)

\[ A_S = \int_{-1.78}^{1.78} f(x) - g(x) \, dx \]

\[ A_S = \int_{-1.78}^{1.78} 4 + \sin(\pi x) - 4^{-x} \, dx \]

\[ A_S = 0.410 \text{ u}^2 \]

Work for problem 1(c)

\[ R = f(x) \quad -1 \leq x \leq 1 \]
\[ r = g(x) \quad -1 \leq x \leq 1 \]

\[ V_S = \int_{-1.78}^{1.78} \pi R^2 - \pi r^2 \, dx \]

\[ V_S = \int_{-1.78}^{1.78} \pi \left( \frac{4}{3} + \sin(\pi x) + 1 \right)^2 - \pi \left( 4^{-x} + 1 \right)^2 \, dx \]

\[ V_S = 4.559 \text{ u}^3 \]
CALCULUS AB
SECTION II, Part A
Time—45 minutes
Number of problems—3

A graphing calculator is required for some problems or parts of problems.

\[ \int_{1.782}^{1.941} (g(x) - f(x)) \, dx \]

= 0.65

Continue problem 1 on page 5.
Work for problem 1(b)

\[
\int_{1.94167}^{1.17821865} f(x) - g(x) \, dx
\]

\[= 1.17\]

Work for problem 1(c)

\[
\int_{1.94167}^{1.17821865} (-1 - f(x))^2 - (-1 - g(x))^2 \, dx
\]

\[= 6.18\]
A graphing calculator is required for some problems or parts of problems.

Work for problem 1(a)

\[ \int_{0}^{\frac{\pi}{4}} - \left[ \frac{1}{4} + \sin(\pi x) \right] \, dx \]

Area \( A = -0.2824 \) units\(^2\)

Continue problem 1 on page 5.
Work for problem 1(b)

\[
\int_{1}^{1} f(x) - g(x) = \int_{1}^{1} \left[ \frac{1}{4} + \sin(\pi x) \right] - 4^{-x}
\]

Area of S = 0.0632 units² via TI-83

Work for problem 1(c)

\[
\pi \int_{-1}^{1} \left( f(x) - g(x) \right)^2 \, dx
\]

\[
\pi \int_{-1}^{1} \left( \frac{1}{4} \sin(\pi x) \right)^2 - 4^{-2x} \, dx
\]

Volume = -14.5243 units³

GO ON TO THE NEXT PAGE.
Work for problem 2(a)

Polar Area = $\frac{1}{2} \int r^2 \, d\theta$

\[ \int_{0}^{\pi} (\theta + \sin(2\theta))^2 \, d\theta = 4.382 \text{ units}^2 \]

Work for problem 2(b)

$\theta + \sin 2\theta = r = \frac{x}{\cos \theta}$

$x = -2$

$\theta + \sin 2\theta = -\frac{2}{\cos \theta}$

$\theta = 2.798$ radians

$\cos \theta = x$

$r = \frac{x}{\cos \theta}$

Continue problem 2 on page 7.
Work for problem 2(c)

This means that the length $r$ is decreasing with respect to $\theta$. Therefore, the distance of the graph from the origin is getting smaller from $\frac{\pi}{3}$ to $\frac{2\pi}{3}$.

Work for problem 2(d)

trying to maximize $r$

$r'(\theta) = 1 + 2 \cos(2\theta)$

$r'(\theta) = 0$ when $\theta = \frac{\pi}{3}$

$r$ is increasing from $0 < \theta < \frac{\pi}{3}$ since $r'(\theta) > 0$

$r$ is decreasing after $\frac{\pi}{2} < \theta < \frac{\pi}{3}$ since $r'(\theta) < 0$

so there is a max at $\theta = \frac{\pi}{3}$
Work for problem 2(a)

The shown area is from $0 \leq \theta \leq \pi$, so the area is given by:

$$\text{area} = \int_{0}^{\pi} \frac{1}{2} (\theta + \sin(2\theta))^2 \, d\theta = 4.382$$

Work for problem 2(b)

$$x = r \cos \theta$$

$$= (\theta + \sin(2\theta))(\cos \theta)$$

Therefore, if $x = 2$,

$$2 = (\theta + \sin(2\theta))(\cos \theta)$$

$$\therefore \theta = 2.786$$
Work for problem 2(c)

Since \( \frac{\pi}{3} < \theta < \frac{2\pi}{3} \), \( \frac{dr}{d\theta} < 0 \), \( r \) is steadily decreasing in that range. Since this range is between 0 and \( \pi \), a range in which a constant-\( r \) function will be concave down, and since this decreases even more than that does because \( r \) is itself diminishing, the curve is concave down in that range.

---

Work for problem 2(d)

If a point has the greatest distance from the origin, \( r \) must be at a maximum. For \( r = \theta + \sin(2\theta) \) from \( 0 \leq \theta \leq \frac{\pi}{2} \), \( r \) reaches a maximum when \( \theta \) is \( \boxed{1.047} \). At that angle, the curve \( \theta \) is thus its furthest from the origin in the first quadrant.
Work for problem 2(a)

\[ A = \frac{1}{2} \int_{0}^{\pi} (\theta + \sin(2\theta))^2 \, d\theta \]

\[ = 8.745 \]

Work for problem 2(b)

\[ x = r \cos \theta \]

\[ -2 = r \cos \theta \]

Continue problem 2 on page 7.
Work for problem 2(c)

This says that as \( \theta \) increases during the interval \( \frac{\pi}{3} + \frac{2\pi}{3} \), the length of \( r \) is decreasing.

It says the curve is bending towards itself more. Because \( r \) is decreasing, the curve is forming a tighter loop which also decreases area.

Work for problem 2(d)

\[ r = \theta + 3\sin(2\theta) \]

\[ \frac{dr}{d\theta} = 1 + 2\cos(2\theta) \]

\[ \theta = 1 - 2\cos(2\theta) \]

\[ \theta = 1.047 \]

\[ \frac{dr}{d\theta} + \rightarrow \]

\[ \frac{dr}{d\theta} \rightarrow - \]

maximum occurs \( \theta = 1.047 \)

because \( \frac{dr}{d\theta} \) changes from positive to negative.

A max. value of \( R \) occurs at \( \theta = 1.047 \)
### Work for problem 3(a)

\[ T'(x) = \frac{55 - 62}{8 - 6} = -3.5 \, ^\circ\text{C/cm} \]

### Work for problem 3(b)

\[
\text{Average Temp} = \frac{1}{8} \int_0^8 T(x) \, dx
\]

\[
\text{Average} = \frac{1}{8} \cdot \left[ (100 + 93)(1)(\frac{1}{2}) + (93 + 70)(4)(\frac{1}{2}) \right. \\
\left. + (62 + 70)(1)(\frac{1}{2}) + (55 + 62)(2)(\frac{1}{2}) \right] \\
= 75.638 \, ^\circ\text{C}
\]

Continue problem 3 on page 9.
Work for problem 3(c)

\[
\int_0^8 T'(x) \, dx = \int (8) - T(0)
\]
\[
= 55 - 100
\]
\[
= -45 \degree C
\]

\[\int_0^8 T'(x) \, dx \text{ mean the total change (drop) in temperature of the wire from 0 cm to 8 cm.}\]

Work for problem 3(d)

\[T''(x) > 0 \Rightarrow T'(x) \text{ is increasing over the period.}\]

\[\text{from } x = 0 \text{ to } 1\]
\[\text{slope } \Rightarrow -7\]

\[x = 1 \text{ to } 5\]
\[\text{slope } \Rightarrow \frac{70 - 93}{5 - 1} = -5.75\]

\[x = 5 \text{ to } 6\]
\[\text{slope } \Rightarrow \frac{62 - 70}{6 - 5} = -8\]

\[x = 6 \text{ to } 8\]
\[\text{slope } \Rightarrow \frac{55 - 62}{8 - 6} = -3.5\]

\[\text{By MVT, there is a point with slope -8 between 5 to 6 which means a decrease of } T'(x)\]

\[T''(x) \text{ is not always increasing}\]

END OF PART A OF SECTION II

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY. DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.
<table>
<thead>
<tr>
<th>Distance $x$ (cm)</th>
<th>0</th>
<th>1</th>
<th>5</th>
<th>6</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temperature $T(x)$ (°C)</td>
<td>100</td>
<td>93</td>
<td>70</td>
<td>62</td>
<td>55</td>
</tr>
</tbody>
</table>

Work for problem 3(a)

$$T'(7) = \frac{T(8) - T(6)}{8 - 6}$$

$$= \frac{55 - 62}{2}$$

$$= -\frac{7}{2} \; °C/cm$$

Work for problem 3(b)

$$\text{Avg}_T = \frac{1}{8} \int_0^8 T(x) \, dx$$

$$\approx \frac{1}{8} \left[ \frac{1}{2}(T(0) + T(1)) + \frac{1}{2}(T(1) + T(5)) + \frac{1}{2}(T(5) + T(6)) + \frac{1}{2}(T(6) + T(8)) \right]$$

$$\approx \frac{1}{8} \left[ \frac{1}{2}(100 + 93) + \frac{1}{2}(93 + 70) + \frac{1}{2}(70 + 62) + \frac{1}{2}(62 + 55) \right]$$

$$\approx 37.813 \; °C$$

Continue problem 3 on page 9.
Work for problem 3(c)

\[ \int_0^8 T'(x) \, dx = T(8) - T(0) \]
\[ = 55 - 100 \]
\[ = -45 \degree C \]

This is the total change in temperature of the wire, from one end to the other.

Work for problem 3(d)

\[ T'(0.5) \equiv \frac{T(0.5) - T(0)}{1 - 0} \]
\[ \equiv -7.00 \]

\[ T'(3) \equiv \frac{T(3) - T(1)}{5 - 1} \]
\[ \equiv -5.750 \]

\[ T'(7) \equiv -3.500 \]

The table is consistent with the assertion that \( T''(x) > 0 \) for every \( x \) in the interval \( 0 < x < 8 \), since \( T'(x) \) is increasing.

END OF PART A OF SECTION II

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY. DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.
Work for problem 3(a)

\[ T'(7) \approx \frac{T(8) - T(6)}{8 - 6} \]
\[ \approx \frac{55 - 62}{2} \]
\[ \approx -\frac{7}{2} \degree C/cm \]

Work for problem 3(b)

\[ \text{Avg} \ T = \frac{\int_0^8 T(x) \, dx}{8} \]

\[ \int_0^8 T(x) \, dx \approx \frac{b-a}{2n} \left[ f(0) + 2f(1) + 2f(5) + 2f(6) + f(8) \right] \]
\[ \approx \frac{8}{2(4)} \left[ 100 + 2(93) + 2(70) + 2(62) + 55 \right] \]
\[ \approx 605 \]

\[ \frac{\int_0^8 T(x) \, dx}{8} \approx \frac{605}{8} = 75.625 \degree C \]

Continue problem 3 on page 9.
Work for problem 3(c)
\[
\int_0^8 T'(x)dx = \left[ T(x) \right]_0^8 = T(8) - T(0) = 55 - 100 = -45 \, ^\circ C/cm
\]
\[\int_0^8 T'(x)dx\] represents the average rate of change of the temperature of the wire as \( x \) increases from 0 to 8.

Work for problem 3(d)

\[ T''(x) > 0 \] implies that \( T(x) \) is concave up, or that the rate of change is increasing.

The data in the table do not show that the rate of change is increasing from \( x = 0 \) to \( x = 8 \). For example: from \( x = 0 \) to \( x = 1 \), \( T(x) \) decreases 7\(^\circ\)C. In order for \( T(x) \) to be concave up, it must decrease by less than 7\(^\circ\)C/cm

from \( x = 1 \) to \( x = 5 \):
\[
\frac{T(5) - T(1)}{4} = -5.75 \, ^\circ C.
\]

\( T(x) \) decreases by 5.75\(^\circ\)C, which less than 7\(^\circ\)C, so it is changing at an increasing rate.

Therefore \( T(x) \) is concave up and \( T''(x) > 0 \) is true for \( x = 0 \) to \( x = 5 \)

\[
\frac{T(6) - T(5)}{1} = -10
\]

This is not consistent however, so \( T(x) \) is not concave up for all \( x \) in \( 0 < x < 8 \).

END OF PART A OF SECTION II

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY. DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.
Work for problem 4(a)

Work for problem 4(b)

Local extrema: $\frac{dy}{dx} = 0$

$0 = 2 \ln\left(\frac{x}{2}\right) - y$

$y = 2 \ln\left(\frac{2}{x}\right)$
Work for problem 4(c)

<table>
<thead>
<tr>
<th>(x, y)</th>
<th>Δx</th>
<th>Δy = Δx * (2x - y)</th>
<th>next (x, y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0, 1)</td>
<td>-0.2</td>
<td>-0.2(-1) = 0.2</td>
<td>(-0.2, 1.2)</td>
</tr>
<tr>
<td>(-0.2, 1.2)</td>
<td>-0.2</td>
<td>-0.2(-0.4-1.2) = 0.32</td>
<td>(-0.4, 1.52)</td>
</tr>
</tbody>
</table>

\[ f(-0.4) = 1.52 \]

Work for problem 4(d)

\[ \frac{d^2y}{dx^2} = 2 - \frac{dy}{dx} = 2 - 2x + y \]

\[ x \text{ negative and } y \text{ positive for values in question; } \]

\[ \frac{d^2y}{dx^2} \text{ positive, graph concave up } \]

The estimation is less than the actual value. The slopes used were greater than the actual slopes; when working from right to left, this means a lesser value.
Work for problem 4(a)

Work for problem 4(b)

\[
\frac{dy}{dx} = 2 \left( \ln \left( \frac{3}{2} \right) \right) - y
\]

\[
2 \left( \ln \left( \frac{3}{2} \right) \right) - y = 0
\]

\[-y = -2 \left( \ln \left( \frac{3}{2} \right) \right)
\]

\[y = 2 \left( \ln \left( \frac{3}{2} \right) \right)\]
Work for problem 4(c)

\( (0,1) \)
\[
\frac{dy}{dx} = 2x - y
\]
\[
\Delta x = -0.2
\]
\[
\Delta y = -1(-0.2) = 0.2
\]
\[
\frac{dy}{dx} = -1
\]

\( (-0.2, 1.2) \)
\[
2(-0.2) - 1.2
\]
\[
\Delta y = 0.8(-0.2) = \frac{0.2}{x = 0.8}
\]
\[
-1.16
\]

\( (-0.4, 1.04) \)
\[
f(-0.4) = 1.04
\]

Work for problem 4(d)

\[
\frac{d^2y}{dx^2} = 2 - \frac{dy}{dx}
\]
\[
\frac{d^2y}{dx^2} = 2 - (2x - y)
\]
\[
\frac{d^2y}{dx^2} = 2 - 2x - y
\]
Work for problem 4(a)

No calculator is allowed for these problems.

Work for problem 4(b)

\[ \frac{dy}{dx} = 2x - 5 \]

\[ \int \frac{1}{5} \, dx = \int 2x \, dx \]

\[ -12 - 5 = e^2 - 1 \ln 1 \]

\[ -12 - 5 = (\ln 3)^2 - 1 \ln 1 \]

\[ -12 - 5 = \ln \frac{e^5}{4} \]

\[ -12 - 5 = \ln \frac{e^5}{4} \]

\[ -\frac{e^5}{4} = x \]

Continue problem 4 on page 11.
Work for problem 4(c)

\[
\begin{align*}
2x - 3 &= \frac{dy}{dx} \\
2(0) - 1 &= \frac{dy}{dx} \\
2(-2) - 1 &= \frac{dy}{dx} \\
-4 &= \frac{dy}{dx}
\end{align*}
\]

Work for problem 4(d)

\[
\begin{align*}
\frac{d^2y}{dx^2} &= z - 5 \\
\frac{d^2y}{dx^2} &= 2 - 2x - y
\end{align*}
\]

Less because the graph is concave up for \((-4, 0)\) which means it is underestimated from the tangent line at \(y\) it'll be less than actual.
Work for problem 5(a)

\[
\int_0^{24} v(t) \, dt = \frac{1}{2} (12+24)(20) = 360 \text{ meters}
\]

\[
\int_0^{24} v(t) \, dt \text{ is the displacement of the car in meters from time } t = 0 \text{ seconds to } t = 24 \text{ seconds. Since the integral is positive, the car is 360 meters in the positive direction at time 24 seconds as compared with its position at time 0 seconds.}
\]

Work for problem 5(b)

\[ v'(4) \text{ does not exist because } \lim_{x \to 4^-} v'(x) = 5 \neq 0 = \lim_{x \to 4^+} v'(x) \]

\[ v'(20) = \frac{-20}{8} = \frac{5}{2} \text{ meters/second}^2 \]

Continue problem 5 on page 13.
Work for problem 5(c)

\[ a(t) = \begin{cases} 
5 \text{ m/s}^2 & \text{for } 0 \leq t < 4 \text{ seconds} \\
0 \text{ m/s}^2 & \text{for } 4 \leq t < 16 \text{ seconds} \\
-5 \text{ m/s}^2 & \text{for } 16 \leq t \leq 24 \text{ seconds} 
\end{cases} \]

* \( a(t) \) undefined for \( t = 4, 16 \text{ seconds} \)

Work for problem 5(d)

\[
\frac{10-20}{20-8} = \frac{-10}{12} = \frac{-5}{6} \text{ m/s}^2
\]

The Mean Value Theorem does not guarantee a value for \( c, \ 8 < c < 20, \)

such that \( v'(c) \) equals this average rate of change because \( v(t) \)

does not fulfill the requirements for the Mean Value Theorem since \( v(t) \)

is not differentiable on the interval \((8, 20)\) since \( v'(t) \) is undefined

at \( t = 16 \text{ seconds} \)
Work for problem 5(a)

\[
\frac{1}{2} (4)(20) + 12(20) + \frac{1}{2} (8)(20) = 40 + 240 + 80 = 360 = \int_{0}^{24} v(t) \, dt
\]

\[
\int_{0}^{24} v(t) \, dt = 360 \text{ meters which is the total distance the car traveled from } t=0 \text{ to } t=24.
\]

Work for problem 5(b)

The derivative of \( v \) at \( t=4 \) does not exist because it is located at a corner.

\[
v'(20) = \frac{20 - 0}{16 - 24} = \frac{20}{-8} = -\frac{5}{2} \text{ meters/sec}
\]
Work for problem 5(c)

\[
\text{slope of } v(t) \text{ from } t=0 \text{ to } t=4 : 5 \\
\text{slope from } t=4 \text{ to } t=16 : 0 \\
\text{slope from } t=16 \text{ to } t=24 : -\frac{5}{2}
\]

\[
a(t) = \begin{cases} 
5 , & 0 \leq x \leq 4 \\
0 , & 4 < x \leq 16 \\
-\frac{5}{2}, & 16 < x \leq 24
\end{cases}
\]

Work for problem 5(d)

The Mean Value Theorem does not guarantee this because \( v(t) \) is not differentiable over \( 8 \leq t \leq 20 \).

\[
\int_{10}^{20} v(t) \, dt = \frac{(10)(20)}{12}
\]

\[
4(20) + 4(20) + \frac{1}{2} (4)(20+10) = 80 + 80 + 60 = 220
\]

\[
\frac{220}{12} \text{ meters/sec}^2
\]
Work for problem 5(a)

\[ \int_0^{24} (v(t)) \, dt \] is asking for the total distance traveled in these 24 seconds. This is the total area underneath the graph.

\[ A = \frac{1}{2} (4)(20) + 12(20) + \frac{1}{2} (8)(20) \]

\[ = 40 + 240 + 80 \]

\[ = 360 \]

Work for problem 5(b)

\[ v'(t) \] is asking for the slope at a certain point in time. At \( t = 4 \) and \( t = 20 \), the slope is undefined because there is a corner at both of these times.

Continue problem 5 on page 13.
Work for problem 5(c)

\[ A(t) = v'(t) \]

\[ v(t) = \begin{cases} 
5x & 0 \leq x \leq 4 \\
-50 & 4 < x \leq 16 \\
-5/2x + 60 & 16 < x \leq 24 
\end{cases} \]

\[ A(t) = \begin{cases} 
5 & 0 \leq x \leq 4 \\
0 & 4 < x \leq 16 \\
-5/2 & 16 < x \leq 24 
\end{cases} \]

\[ \text{Note:} \ 5 \text{ is the derivative of } 5x \]

\[ \text{Note:} \ 0 \text{ is the derivative of } 20 \]

\[ \text{Note:} \ -5/2 \text{ is the derivative of } -5/2x + 60 \]

Work for problem 5(d)

Average rate of change is:

\[ \frac{f(b) - f(a)}{b - a} = \frac{f(20) - f(0)}{20 - 0} = \frac{10 - 20}{20} = \frac{-10}{20} = \frac{-5}{10} \]

Yes, c has a guaranteed value because \( v(t) \) is continuous from \( 0 \leq x \leq 20 \).
Work for problem 6(a)

\[ f'(2) = 7 \]
\[ f''(2) = f'''(2) = f''''(2) = 0 \]
\[ n^{(2)} = \frac{(2-1)!}{3^2} = \frac{1}{9} \]
\[ n^{(4)}(2) = \frac{(4-1)!}{3^4} = \frac{6}{81} = \frac{2}{27} \]
\[ n^{(4)}(2) = \frac{(6-1)!}{3^6} = \frac{5 \cdot 4 \cdot 3 \cdot 2}{3^6} = \frac{40}{243} \]

\[ P_n(x) = 7 + \frac{1}{9} (x-2)^2 + \frac{2}{27} (x-2)^4 + \frac{40/243}{6!} (x-2)^6 \]

Work for problem 6(b)

coefficient = \( \frac{(2n-1)!}{3^n \cdot (2n)!} = \frac{(2n-1)!}{(3 \cdot 2n)!} = \frac{1}{2n(3^n)} \)

Continue problem 6 on page 15.
NO CALCULATOR ALLOWED

Work for problem 6(c)

\[
1 + \sum_{n=1}^{\infty} \frac{(x-2)^{2n}}{2n \cdot (9^n)}
\]

\[
\lim_{n \to \infty} \left| \frac{(x-2)^{2n}}{2n+2 \cdot (q^{n+1})} \cdot \frac{2n \cdot (9^n)}{(x-2)^{2n}} \right|
\]

\[
\lim_{n \to \infty} \left\{ \frac{2n}{2n+2} \cdot \frac{(x-2)^2}{q} \right\}
\]

\[
\left| \frac{(x-2)^2}{q} \right| < 1
\]

\[
(x-2)^2 < 9
\]

-3 < x-2 < 3

-1 < x < 5

If \( x = -1 \):
\[
\sum_{n=1}^{\infty} \frac{(-3)^{2n}}{2n \cdot (3^n)} = \sum_{n=1}^{\infty} \frac{(-1)^{2n}}{2n} = \sum_{n=1}^{\infty} \frac{1}{2n}
\]

\( \frac{1}{n} \) = harmonic series
\( \therefore \) DN

If \( x = 5 \):
\[
\sum_{n=1}^{\infty} \frac{3^{2n}}{2n \cdot (3^n)} = \sum_{n=1}^{\infty} \frac{3^n}{2 \cdot n}
\]

harmonic series
\( \therefore \) DN

END OF EXAM

THE FOLLOWING INSTRUCTIONS APPLY TO THE BACK COVER OF THIS SECTION II BOOKLET.

- MAKE SURE YOU HAVE COMPLETED THE IDENTIFICATION INFORMATION AS REQUESTED ON THE BACK OF THIS SECTION II BOOKLET.
- CHECK TO SEE THAT YOUR AP NUMBER APPEARS IN THE BOX(ES) ON THE BACK COVER.
- MAKE SURE THAT YOU HAVE USED THE SAME SET OF AP NUMBER LABELS ON ALL AP EXAMS YOU HAVE TAKEN THIS YEAR.
Work for problem 6(a)

\[ P_{6}(x) = 7 + (x-2)(f'(2)) + \frac{1}{2!} (x-2)^2 \]

\[ P_{6}(x) = 7 + \frac{11(x-2)^2}{9 \cdot 2!} + \frac{3! (x-2)^4}{3^4 \cdot 4!} + \frac{5! (x-2)^6}{3^6 \cdot 6!} \]

\[ P_{8}(x) = 7 + \frac{(x-2)^2}{9 \cdot 2!} + \frac{(x-2)^4}{3^4 \cdot 4!} + \frac{(x-2)^6}{3^6 \cdot 6!} \]

Work for problem 6(b)

\[ \frac{(x-2)^{2n}}{3 \cdot (2n)} \]

\[ n \geq 1 \]

Coefficient is \[ \frac{1}{3 \cdot (2n)} \]
Work for problem 6(c)

\[
7 + \sum_{n=1}^{\infty} \frac{(x-2)^{2n}}{3^{2n}(2n)}
\]

Left hand: \( x = 2 \)

\[
\sum_{n=1}^{\infty} \frac{0}{3^{2n}(2n)} = 0 \quad \text{converge}
\]

Right hand: \( x = 5 \)

\[
\sum_{n=1}^{\infty} \frac{(3)^{2n}}{3^{2n}(2n)} = \frac{1}{2n} \quad \text{harm. series diverge}
\]

therefore \( \sum_{n=1}^{\infty} \frac{(x-2)^{2n}}{3^{2n}(2n)} \) has interval of convergence of \((2, 5)\)

\[
\lim_{a_n \to \infty} \left| \frac{(x-2)^{2n+2}}{3^{2n+2}(2n+2)} \cdot \frac{3^{2n}(2n)}{(x-2)^{2n}} \right| = 1
\]

\[
\lim_{a_n \to \infty} \left( \frac{(x-2)^2}{9(2n+2)} \right) < 1
\]

\(-1 < \frac{(x-2)^2}{9} < 1\)

but some has to be \(0\)

\(-9 < (x-2)^2 < 9\)

\[0 < x-2 < 3\]

\[2 < x < 5\]

\[Q < x < 5\]

THE FOLLOWING INSTRUCTIONS APPLY TO THE BACK COVER OF THIS SECTION II BOOKLET.

- MAKE SURE YOU HAVE COMPLETED THE IDENTIFICATION INFORMATION AS REQUESTED ON THE BACK OF THIS SECTION II BOOKLET.
- CHECK TO SEE THAT YOUR AP NUMBER APPEARS IN THE BOX(ES) ON THE BACK COVER.
- MAKE SURE THAT YOU HAVE USED THE SAME SET OF AP NUMBER LABELS ON ALL AP EXAMS YOU HAVE TAKEN THIS YEAR.
Work for problem 6(a)

\[ f(2) = 7, \]

1st, 3rd, 5th derivatives = 0.

\[ f'(2) = 0, \quad f''(2) = \frac{1}{9}, \quad f^4(2) = \frac{1}{3^4}, \quad f^6(2) = \frac{5}{3^6} \]

Taylor Series

\[ f(x) = f(2) + f'(2)(x-2) + \frac{f''(2)}{2!}(x-2)^2 + \frac{f^4(2)}{3!}(x-2)^3 + \ldots \]

\[ = 7 + 0 + \frac{\frac{1}{9}}{2!}(x-2)^2 + 0 + \frac{\frac{1}{3^4}}{3!}(x-2)^4 + 0 + \frac{\frac{5}{3^6}}{6!}(x-2)^6 \]

\[ = 7 + \frac{1}{18}(x-2)^2 + \frac{(x-2)^4}{3^4 \cdot 4} + \frac{(x-2)^6}{3^6 \cdot 6} \]

Work for problem 6(b)

General term for Taylor series about \( x = 2 \)

\[ \frac{(x-2)^{2n}}{3^{2n} \cdot n} \]

The coefficient of \( (x-2)^{2n} \) for \( n \geq 1 \) is

0 when \( n \) is odd, and \( \frac{1}{3^{2n} \cdot n} \) when \( n \) is even and \( n \geq 2 \).

Continue problem 6 on page 15.
Work for problem 6(c)

\[
\lim_{n \to \infty} \left| \frac{(x-2)^2 n}{3^{2n}(n+1)} \right| = \lim_{n \to \infty} \frac{(x-2)^2 n}{3^{2n}(n+1)}
\]

\[
\lim_{n \to \infty} \frac{3^{2n} n}{3^{2n}(n+1)} \to \infty
\]

\[
\text{as } n \to \infty, \text{ the}
\]