Question 1

(a) \[ \int_{0}^{300} r(t) \, dt = 270 \]

According to the model, 270 people enter the line for the escalator during the time interval \( 0 \leq t \leq 300 \).

(b) \[ 20 + \int_{0}^{300} (r(t) - 0.7) \, dt = 20 + \int_{0}^{300} r(t) \, dt - 0.7 \cdot 300 = 80 \]

According to the model, 80 people are in line at time \( t = 300 \).

(c) Based on part (b), the number of people in line at time \( t = 300 \) is 80.

The first time \( t \) that there are no people in line is \( 300 + \frac{80}{0.7} = 414.286 \) (or 414.285) seconds.

(d) The total number of people in line at time \( t \), \( 0 \leq t \leq 300 \), is modeled by \( 20 + \int_{0}^{t} r(x) \, dx - 0.7t \).

\[ r(t) - 0.7 = 0 \Rightarrow t_1 = 33.013298, \ t_2 = 166.574719 \]

<table>
<thead>
<tr>
<th>( t )</th>
<th>People in line for escalator</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>20</td>
</tr>
<tr>
<td>( t_1 )</td>
<td>3.803</td>
</tr>
<tr>
<td>( t_2 )</td>
<td>158.070</td>
</tr>
<tr>
<td>300</td>
<td>80</td>
</tr>
</tbody>
</table>

The number of people in line is a minimum at time \( t = 33.013 \) seconds, when there are 4 people in line.
Question 2

(a) \(v'(3) = -2.118\)

The acceleration of the particle at time \(t = 3\) is \(-2.118\).

(b) \(x(3) = x(0) + \int_0^3 v(t) \, dt = -5 + \int_0^3 v(t) \, dt = -1.760213\)

The position of the particle at time \(t = 3\) is \(-1.760\).

(c) \(\int_0^{3.5} v(t) \, dt = 2.844\) (or 2.843)

\(\int_0^{3.5} |v(t)| \, dt = 3.737\)

The integral \(\int_0^{3.5} v(t) \, dt\) is the displacement of the particle over the time interval \(0 \leq t \leq 3.5\).

The integral \(\int_0^{3.5} |v(t)| \, dt\) is the total distance traveled by the particle over the time interval \(0 \leq t \leq 3.5\).

(d) \(v(t) = x_2'(t)\)

\[v(t) = 2t - 1 \implies t = 1.57054\]

The two particles are moving with the same velocity at time \(t = 1.571\) (or 1.570).
(a) \[ f(-5) = f(1) + \int_{1}^{-5} g(x) \, dx = f(1) - \int_{-5}^{1} g(x) \, dx \]
\[ = 3 - \left( -9 - \frac{3}{2} + 1 \right) = 3 - \left( -\frac{19}{2} \right) = \frac{25}{2} \]

(b) \[ \int_{1}^{6} g(x) \, dx = \int_{1}^{3} g(x) \, dx + \int_{3}^{6} g(x) \, dx \]
\[ = \int_{1}^{3} 2 \, dx + \int_{3}^{6} (2x - 4)^2 \, dx \]
\[ = 4 + \left[ \frac{2}{3} (x - 4)^3 \right]_{x=3}^{x=6} = 4 + \frac{16}{3} - \left( -\frac{2}{3} \right) = 10 \]

(c) The graph of \( f \) is increasing and concave up on \( 0 < x < 1 \) and \( 4 < x < 6 \) because \( f'(x) = g(x) > 0 \) and \( f''(x) = g(x) \) is increasing on those intervals.

(d) The graph of \( f \) has a point of inflection at \( x = 4 \) because \( f''(x) = g(x) \) changes from decreasing to increasing at \( x = 4 \).
Question 4

(a) \( H'(6) \approx \frac{H(7) - H(5)}{7 - 5} = \frac{11 - 6}{2} = \frac{5}{2} \)

\( H'(6) \) is the rate at which the height of the tree is changing, in meters per year, at time \( t = 6 \) years.

(b) \( \frac{H(5) - H(3)}{5 - 3} = \frac{6 - 2}{2} = 2 \)

Because \( H \) is differentiable on \( 3 \leq t \leq 5 \), \( H \) is continuous on \( 3 \leq t \leq 5 \).

By the Mean Value Theorem, there exists a value \( c \), \( 3 < c < 5 \), such that \( H'(c) = 2 \).

(c) The average height of the tree over the time interval \( 2 \leq t \leq 10 \) is given by \( \frac{1}{10 - 2} \int_2^{10} H(t) \, dt \).

\[
\frac{1}{8} \int_2^{10} H(t) \, dt \approx \frac{1}{8} \left( \frac{1.5}{2} \cdot 1 + \frac{2 + 6}{2} \cdot 2 + \frac{6 + 11}{2} \cdot 2 + \frac{11 + 15}{2} \cdot \frac{3}{2} \right) \\
= \frac{1}{8} (65.75) = \frac{263}{32}
\]

The average height of the tree over the time interval \( 2 \leq t \leq 10 \) is \( \frac{263}{32} \) meters.

(d) \( G(x) = 50 \Rightarrow x = 1 \)

\[
\frac{d}{dt}(G(x)) = \frac{d}{dx}(G(x)) \cdot \frac{dx}{dt} = \frac{(1 + x)100 - 100x \cdot 1}{(1 + x)^2} \cdot \frac{dx}{dt} = \frac{100}{(1 + x)^2} \cdot \frac{dx}{dt}
\]

\[
\frac{d}{dt}(G(x)) \bigg|_{x=1} = \frac{100}{(1 + 1)^2} \cdot 0.03 = \frac{3}{4}
\]

According to the model, the rate of change of the height of the tree with respect to time when the tree is 50 meters tall is \( \frac{3}{4} \) meter per year.
Question 5

(a) The average rate of change of \( f \) on the interval \( 0 \leq x \leq \pi \) is
\[
\frac{f(\pi) - f(0)}{\pi - 0} = -e^\pi - 1.
\]

(b) \( f'(x) = e^x \cos x - e^x \sin x \)
\[
f'(\frac{3\pi}{2}) = e^{3\pi/2} \cos\left(\frac{3\pi}{2}\right) - e^{3\pi/2} \sin\left(\frac{3\pi}{2}\right) = e^{3\pi/2}
\]

The slope of the line tangent to the graph of \( f \) at \( x = \frac{3\pi}{2} \) is \( e^{3\pi/2} \).

(c) \( f'(x) = 0 \Rightarrow \cos x - \sin x = 0 \Rightarrow x = \frac{\pi}{4}, x = \frac{5\pi}{4} \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>( \frac{\pi}{4} )</td>
<td>( \frac{1}{\sqrt{2}} e^{\pi/4} )</td>
</tr>
<tr>
<td>( \frac{5\pi}{4} )</td>
<td>( -\frac{1}{\sqrt{2}} e^{5\pi/4} )</td>
</tr>
<tr>
<td>2\pi</td>
<td>( e^{2\pi} )</td>
</tr>
</tbody>
</table>

The absolute minimum value of \( f \) on \( 0 \leq x \leq 2\pi \) is \( -\frac{1}{\sqrt{2}} e^{5\pi/4} \).

(d) \( \lim_{x \to \pi/2} f(x) = 0 \)
Because \( g \) is differentiable, \( g \) is continuous.
\[
\lim_{x \to \pi/2} g(x) = g\left(\frac{\pi}{2}\right) = 0
\]

By L’Hospital’s Rule,
\[
\lim_{x \to \pi/2} f(x) = \lim_{x \to \pi/2} g(x) = \frac{e^{\pi/2}}{2}.
\]
Question 6

(a) Curves must go through the indicated points, follow the given slope lines, and extend to the boundary of the slope field.

(b) \( \frac{dy}{dx} \left|_{(x, y) = (1, 0)} \right. = \frac{4}{3} \)

An equation for the line tangent to the graph of \( y = f(x) \) at \( x = 1 \) is \( y = \frac{4}{3}(x - 1) \).

\[ f(0.7) \approx \frac{4}{3}(0.7 - 1) = -0.4 \]

(c) \( \frac{dy}{dx} = \frac{1}{3} x(y - 2)^2 \)

\[ \int \frac{dy}{(y - 2)^2} = \int \frac{1}{3} x \, dx \]

\[ \frac{-1}{y - 2} = \frac{1}{6} x^2 + C \]

\[ \frac{1}{2} = \frac{1}{6} + C \Rightarrow C = \frac{1}{3} \]

\[ \frac{-1}{y - 2} = \frac{1}{6} x^2 + \frac{1}{3} = \frac{x^2 + 2}{6} \]

\[ y = 2 - \frac{6}{x^2 + 2} \]

Note: this solution is valid for \( -\infty < x < \infty \).