

## Exploring Slope

### High Ratio Mountain

#### Lesson 11-1 Linear Equations and Slope

#### ACTIVITY 11

#### Learning Targets:

- Understand the concept of slope as the ratio  $\frac{\text{change in } y}{\text{change in } x}$  between any two points on a line.
- Graph proportional relationships; interpret the slope and the  $y$ -intercept  $(0, 0)$  of the graph.
- Use similar right triangles to develop an understanding of slope.

**SUGGESTED LEARNING STRATEGIES:** Create Representations, Marking The Text, Discussion Groups, Sharing and Responding, Interactive Word Wall

Misty Flipp worked odd jobs all summer long and saved her money to buy passes to the ski lift at the High Ratio Mountain Ski Resort. In August, Misty researched lift ticket prices and found several options. Since she worked so hard to earn this money, Misty carefully investigated each of her options.

High Ratio Mountain Ski Resort	
Student Lift Ticket prices	
Daily Lift Ticket	\$30
10-Day Package	\$80 upon purchase and \$20 per day (up to 10 days)
Unlimited Season Pass	\$390

- Suppose Misty purchases a daily lift ticket each time she goes skiing. Complete the table below to determine the total cost for lift tickets.

Number of Days	0	1	2	3	4	5	6
Total Cost of Lift Tickets	0	30	60	90	120	150	180

- According to the table, what is the relationship between the cost of the lift tickets and the number of days?  
**As the number of days increases by 1, the total cost increases by \$30.**

#### My Notes

## ACTIVITY 11

### Investigative

#### Activity Standards Focus

This activity deals with the connections between proportional relationships, lines and linear equations. Here students develop their understanding of slope as rate of change and as a ratio. They will graph proportional relationships, determine slope and  $y$ -intercept from graphs, and interpret slope and  $y$ -intercept in the context of real-world and mathematical problems.

#### Lesson 11-1

#### PLAN

**Pacing:** 1–2 class periods

#### Chunking the Lesson

#1–3      #4–6      #7–8  
#9–11    #13      #14

Check Your Understanding

Lesson Practice 11–1

#### TEACH

#### Bell-Ringer Activity

Give students a few minutes to write down all the terms and concepts that come to mind when they hear the word *slope*. Have a few volunteers read their lists aloud. Use any results relating to the ideas of slant or steepness to introduce the concept of the slope of a line.

#### 1–3 Shared Reading, Create Representations, Look for a Pattern, Discussion Groups, Sharing and Responding

Students will use the table, a verbal description and an equation to represent the relationship between the number of days that Misty skis and the total cost of her ski season. These items are designed to move students toward expressing rate of change verbally.

#### Common Core State Standards for Activity 11

- 8.EE.B.5 Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.*
- 8.EE.B.6 Use similar triangles to explain why the slope  $m$  is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation  $y = mx$  for a line through the origin and the equation  $y = mx + b$  for a line intercepting the vertical axis at  $b$ .

4–6 Activating Prior Knowledge, Create Representations, Think-Pair-Share, Sharing and Responding

Students will use the graph and a verbal description to represent the relationship between the number of days that Misty skis and the total cost of her ski season. These items are also designed to move students toward expressing rate of change verbally.

7–8 Look for a Pattern, Discussion Groups, Sharing and Responding

These items have students think about the rate of change through horizontal and vertical movements on the graph. That leads directly into the concept of finding a slope graphically by asking students to write the rate of change as a ratio comparing vertical change to horizontal change.

ACTIVITY 11 continued

My Notes

MATH TIP

Vertical change is the number of spaces moved up or down on a graph. “Up” movement is represented by a positive number. “Down” is a negative number.

Horizontal change is the number of spaces moved right or left on a graph. Movement to the right is indicated by a positive number. Movement to the left is indicated by a negative number.

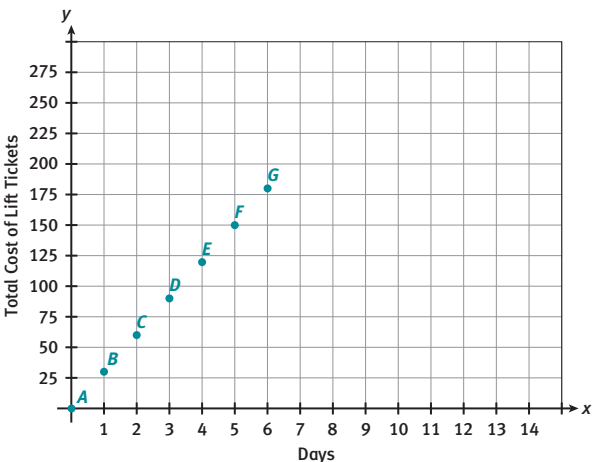
Lesson 11-1 Linear Equations and Slope

3. Let  $d$  represent the number of days for which Misty bought lift tickets and  $C$  represent Misty’s total cost. Write an equation that can be used to determine the total cost of lift tickets if Misty skis for  $d$  days.

$C = 30d$

4. **Model with mathematics.** Plot the data from the table on the graph below. The data points appear to be linear. What do you think this means?

Sample answer: Linear means that the points could be connected to create a line.



5. Label the leftmost point on the graph point A. Label the next 6 points, from left to right, points B, C, D, E, F, and G. **See graph above.**
6. **Reason quantitatively.** According to the graph, what happens to the total cost of lift tickets as the number of days increases? Justify your answer.

As the number of days increases by 1, the total cost increases by \$30. For each space you move right, you move 30 spaces up.

7. Describe the movement, on the graph, from one point to another.
- |         |                 |      |                   |   |
|---------|-----------------|------|-------------------|---|
| A to B: | Vertical Change | \$30 | Horizontal Change | 1 |
| B to C: | Vertical Change | \$30 | Horizontal Change | 1 |
| C to D: | Vertical Change | \$30 | Horizontal Change | 1 |
| D to E: | Vertical Change | \$30 | Horizontal Change | 1 |
| E to F: | Vertical Change | \$30 | Horizontal Change | 1 |
| F to G: | Vertical Change | \$30 | Horizontal Change | 1 |

MINI-LESSON: Ratios

For students who are struggling with ratios, a mini-lesson is available to provide examples and practice. In this lesson, students are given real-world situations that can be represented by ratios and asked to provide the ratios, as well as to express them in different ways: verbally, in fractional notation, and with a semi-colon.

See SpringBoard’s eBook Teacher Resources for a student page for this mini-lesson.

## Lesson 11-1

### Linear Equations and Slope

8. a. The movements you traced in Item 7 can be written as ratios. Write ratios in the form  $\frac{\text{vertical change}}{\text{horizontal change}}$  to describe the movement from:

$$A \text{ to } B: \frac{30}{1}$$

$$B \text{ to } C: \frac{30}{1}$$

$$C \text{ to } D: \frac{30}{1}$$

$$D \text{ to } E: \frac{30}{1}$$

- b. Vertical change can also be described as the *change in y*. Similarly, the horizontal change is often referred to as the *change in x*. Therefore, the ratio  $\frac{\text{vertical change}}{\text{horizontal change}}$  can also be written as  $\frac{\text{change in } y}{\text{change in } x}$ . Determine the *change in y* and *change in x* from A to C in Item 4. Write the ratio as  $\frac{\text{change in } y}{\text{change in } x}$ .

A is (0, 0) and C is (2, 60).

$$\frac{\text{vertical change}}{\text{horizontal change}} = \frac{\text{change in } y}{\text{change in } x} = \frac{0 \text{ to } 60}{0 \text{ to } 2} = \frac{60}{2}$$

Continue to use the data from Item 4. Determine the *change in y* and *change in x* for each movement described below. Then write the ratio  $\frac{\text{change in } y}{\text{change in } x}$ .

- c. From B to E:  $\frac{90}{3}$
- d. From A to E:  $\frac{120}{4}$
- e. From B to A:  $\frac{-30}{-1}$
- f. From E to B:  $\frac{-90}{-3}$

## ACTIVITY 11

### continued

### My Notes

### READING AND WRITING MATH

When writing a ratio, you can also represent the relationship by separating each quantity with a colon. For example, the ratio 1:4 is read "one to four."

## ACTIVITY 11 Continued

**7-8 (continued)** Students use the graph to calculate  $\frac{\text{change in } y}{\text{change in } x}$  ratio between a series of points and discover that the ratio that represents the rate of change is a constant, including calculations for which the *change in y* and *change in x* are negative. Make sure that the students properly distinguish the direction of the *change in y* and *change in x* in each case, rather than simply determining the magnitude of the change.

## ACTIVITY 11 Continued

### 9–11 Shared Reading, Marking the Text, Discussion Groups, Look for a Pattern, Sharing and Responding

These items, are designed to facilitate student understanding of the fact that the rate of change here is constant and to aid students in relating the rate of change to Misty's ski scenario. Students should see that the ratios they write are equivalent, although not all in lowest terms, and understand that the sign of the ratio is not dependent on direction since the division of two negative numbers yields a positive quotient.

### 12 Shared Reading, Marking the Text, Activating Prior Knowledge, Think-Pair-Share

Students use what they know about scale factors and similar triangles to further develop their understanding of a constant rate of change. By using the proportional sides in similar triangles they can see that the ratio  $\frac{\text{change in } y}{\text{change in } x}$  between any two points on a line is constant.

### TEACHER TO TEACHER

This activity discusses slope as a constant rate of change. Students can use this concept to determine slope given a graph, a table or two points. The slope formula itself is confusing to some students.

Having a strong grounding in the concept of slope as a rate of change will make the transition to the formulaic understanding easier for students and be beneficial in higher level mathematics courses such as Algebra II, Precalculus and Calculus.

## ACTIVITY 11

continued

### My Notes

### MATH TIP

In similar triangles, corresponding angles are congruent and corresponding sides are in proportion.

## Lesson 11-1 Linear Equations and Slope

9. Describe the similarities and differences in the ratios written in Item 8. How are the ratios related?

The ratios are all equivalent to  $\frac{30}{1}$ , although they are not all in lowest terms.

10. **Make sense of problems.** What are the units of the ratios created in Item 8? Explain how the ratios and units relate to Misty's situation.

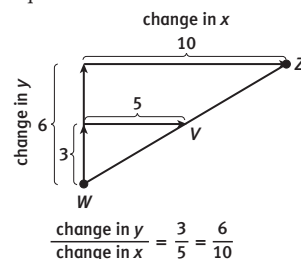
Dollars each day. The ratios show the \$30 increase in price for each day that she skis.

11. How do the ratios relate to the equation you wrote in Item 3?

The ratio  $\frac{30}{1}$  represents the \$30 that is multiplied by the number of days in the equation.

12. The ratio  $\frac{\text{change in } y}{\text{change in } x}$  between any two points on a line is constant.

Use the diagram below and what you know about similar triangles to explain why the  $\frac{\text{change in } y}{\text{change in } x}$  ratios are equivalent for the movements described.



They are equivalent because the triangles are similar and the corresponding sides are being compared.

## Lesson 11-1

### Linear Equations and Slope

The **slope** of a line is determined by the ratio  $\frac{\text{change in } y}{\text{change in } x}$  between any two points that lie on the line.

- The slope is the *constant rate of change* of a line. It is also sometimes called the *average rate of change*.
- All linear relationships have a *constant rate of change*.
- The slope of a line is what determines how steep or flat the line is.
- The  $y$ -intercept of a line is the point at which the line crosses the  $y$ -axis,  $(0, y)$ .

13. Draw a line through the points you graphed in Item 4. Use the graph to determine the slope and  $y$ -intercept of the line. How do the slope and  $y$ -intercept of this line relate to the equation you wrote in Item 3?

**Slope:**  $\frac{30}{1}$ ;  **$y$ -intercept:**  $(0, 0)$ . The slope is the coefficient of  $d$  in the equation.  $(0, 0)$  represents the cost of tickets for 0 days.

14. Complete the table to show the data points you graphed in Item 4. Use the table to indicate the ratio  $\frac{\text{change in } y}{\text{change in } x}$  and to determine the slope of the line.

Number of Days	Total Cost of Lift Tickets
0	0
1	30
2	60
3	90
4	120
5	150
6	180

change in  $y$ : 30      change in  $x$ : 1

$\frac{\text{change in } y}{\text{change in } x}$ :  $\frac{30}{1}$       slope:  $\frac{30}{1}$

## ACTIVITY 11

### continued

### My Notes

### MATH TERMS

**Slope** is the ratio of vertical change to horizontal change, or  $\frac{\text{change in } y}{\text{change in } x}$ .

### READING MATH

The slope of a line,  $\frac{\text{change in } y}{\text{change in } x}$ , is also expressed symbolically as  $\frac{\Delta y}{\Delta x}$ .  $\Delta$  is the Greek letter delta, and in mathematics it means "change in."

## ACTIVITY 11 Continued

### Developing Math Language

This lesson contains several vocabulary terms. The word **slope** is introduced here and related words **linear** and  **$y$ -intercept** are reviewed. Time should be spent to read closely, mark the text and develop the relationships between these terms. Attention should also be drawn to the notation  $\frac{\Delta y}{\Delta x}$ .

As you guide students through their learning of these essential mathematical terms, explain meanings in language that is accessible for your students. Whenever possible, provide concrete examples to help students gain understanding. Encourage students to make notes about new terms and their understanding of what they mean, and about how to use them to describe precise mathematical concepts and processes.

### TEACHER TO TEACHER

Add words like the terms introduced above to your classroom Word Wall regularly. Include math terms, academic vocabulary, and other words that students use regularly in their group or class discussions. To remind students to refer to the Word Wall, ask them to choose words to add. Another way to reinforce language acquisition is to have each student choose a word from the Word Wall and then pair-share for a few minutes to discuss meaning and use.

**13 Create Representations, Look for a Pattern, Discussion Groups, Sharing and Responding** Students identify the slope and  $y$ -intercept of the linear model they create and then explore the relationships between the values they find on the graph and the terms of the equation they created to represent Misty's ski season. The idea here is that students will begin to make connections between the different representations of the data.

**14 Create Representations, Look for a Pattern, Discussion Groups, Sharing and Responding** Students return to what they know about the ratio  $\frac{\text{change in } y}{\text{change in } x}$  and use the rate of change identified in a table to determine the slope of the line in question. Again, the idea here is that students begin to make connections between different representations of the data and the slope.

Check Your Understanding

Debrief this section of the activity by asking students to describe the relationship between slope,  $\frac{\text{change in } y}{\text{change in } x}$  and  $\frac{\Delta y}{\Delta x}$ . How would they use each to determine the rate of change of a linear model from a graph? From a table?

Answers

15. a. slope: 3; y-intercept: (0, 0)  
b. slope:  $-\frac{2}{1}$ ; y-intercept: (0, -1)  
c. slope:  $\frac{2.5}{1}$ ; y-intercept: (0, 0)  
d. slope:  $-\frac{2}{1}$ ; y-intercept: (0, 2)  
e. Yes. Sample explanation:  
From W, extend the change-in-y segment to 9 units and extend the change-in-x segment to 15 units to locate point P and create another right triangle. The ratio of the change in y to the change in x is  $\frac{9}{15}$  or  $\frac{3}{5}$ , the same as the ratios for W and V and for W and Z, so P must be on the same line.
16.  $\frac{\text{change in } y}{\text{change in } x} = \frac{2500 - 1000}{5 - 2} = \frac{1500}{3}$   
John's rate is 500 feet per minute.

ACTIVITY 11  
continued

My Notes

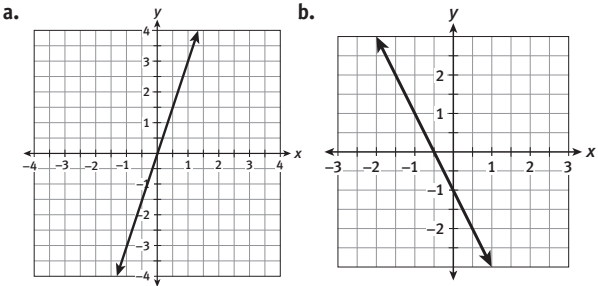
CONNECT TO SPORTS

Longboards are larger than the more trick-oriented skateboards. Longboards are heavier and sturdier than skateboards. Some people even use them instead of bicycles.

Lesson 11-1  
Linear Equations and Slope

Check Your Understanding

15. Find the slope and the y-intercept for each of the following. Remember to use the ratio  $\frac{\text{change in } y}{\text{change in } x}$ .



c.

x	y
0	0
1	2.5
2	5
4	10

d.

x	y
-1	4
0	2
1	0
3	-4

- e. Look back at the figure for Item 12. Would a point P that is 9 units up from point W and 15 units to the right be on the line that contains points W, V, and Z? Use similar triangles to explain your answer.
16. John is longboarding at a constant rate down the road. If 2 minutes after he leaves his house he is 1,000 feet away and at 5 minutes he is 2,500 feet from his house, what would his average rate of change be?

MINI-LESSON: Finding Slope Given a Table or a Graph

Some students may need further practice in determining slope from graphs and/or tables representing linear relationships. For that purpose a mini-lesson has been provided, containing a problem of each kind as an example to show the student. The instruction for the problems suggests working with them in such a way as to emphasize the fact that slope remains constant in a linear relationship.

See SpringBoard's eBook Teacher Resources for a student page for this mini-lesson.



## Lesson 11-1

### Linear Equations and Slope

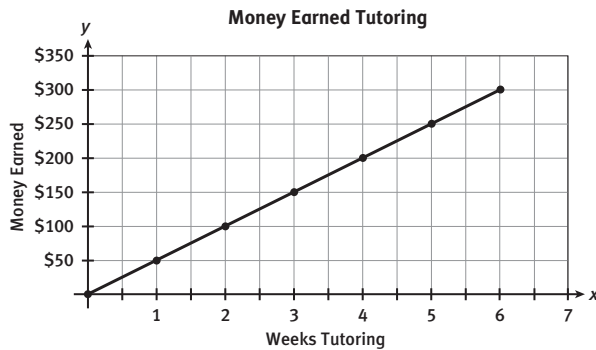
#### LESSON 11-1 PRACTICE

The Tran family is driving across the country. They drive 400 miles each day. Use the table below to answer Items 17–20.

Day	Total Miles Driven
1	400
2	800
3	
4	
5	

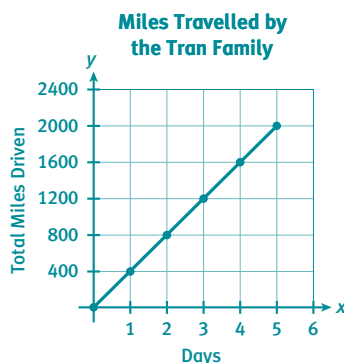
17. Complete the table.
18. Draw a graph for the data in the table. Be sure to title the graph and label the axes. Draw a line through the points.
19. Write an equation that can be used to determine the total miles,  $M$ , driven over  $d$  days.
20. Find the slope and the  $y$ -intercept of the line you created, using the graph you drew or the equation you wrote. Explain what each represents for the Tran family's situation.

The graph below shows the money a student earns as she tutors. Use the graph to answer Items 21–24.



21. What is the slope of the line?
22. What is the  $y$ -intercept of the line?
23. Write an equation that can be used to determine how much money,  $D$ , the student has earned after  $w$  weeks.
24. **Attend to precision.** Calculate how much money the student will have earned after 52 weeks.

18. Miles Travelled by the Tran Family



19.  $M = 400d$
20. The slope is  $\frac{400}{1}$ . The  $y$ -intercept is  $(0, 0)$ . The slope represents how many miles the family drives each day. The  $y$ -intercept is how many miles they had driven at the very start of the trip, or 0.
21.  $\frac{50}{1}$
22.  $(0, 0)$
23.  $D = \$50w$
24.  $\$50 \times 52 \text{ weeks} = \$2600$

## ACTIVITY 11

### continued

#### My Notes

## ACTIVITY 11 Continued

### ASSESS

Students' answers to lesson practice problems will provide you with a formative assessment of their understanding of the lesson concepts and their ability to apply their learning.

See the Activity Practice for additional problems for this lesson. You may assign the problems here or use them as a culmination for the activity.

#### LESSON 11-1 PRACTICE

17.

Days	Total Miles Driven
1	400
2	800
3	1200
4	1600
5	2000

### ADAPT

Some students may need more explicit instruction on how slope and  $y$ -intercept are determined from the various representations of linear relationships (equations, tables and graphs) and what they mean. Such students may benefit by being guided through a problem which moves students back and forth between the different representations of some linear relationship, focusing on slope and  $y$ -intercept at each step of the way and allows them to make connections.

Give students a verbal description of a linear relationship situation, and then demonstrate how to represent the situation using an equation. Work with the students to identify the slope and  $y$ -intercept in the equation. Students may then create a table of values, again highlighting the slope and  $y$ -intercept, and relate the meaning of the slope and  $y$ -intercept to the context of the problem. Finally, they should identify the slope and  $y$ -intercept on a graph of the relationship, while also reiterating the meaning of the terms in the context of the problem. It may be helpful to create a mat that shows the three different representations, along with the slope and  $y$ -intercept highlighted in each.